UNIVERSIDAD NACIONAL DE EDUCACIÓN A DISTANCIA

ANÁLISIS DE ESTRUCTURAS

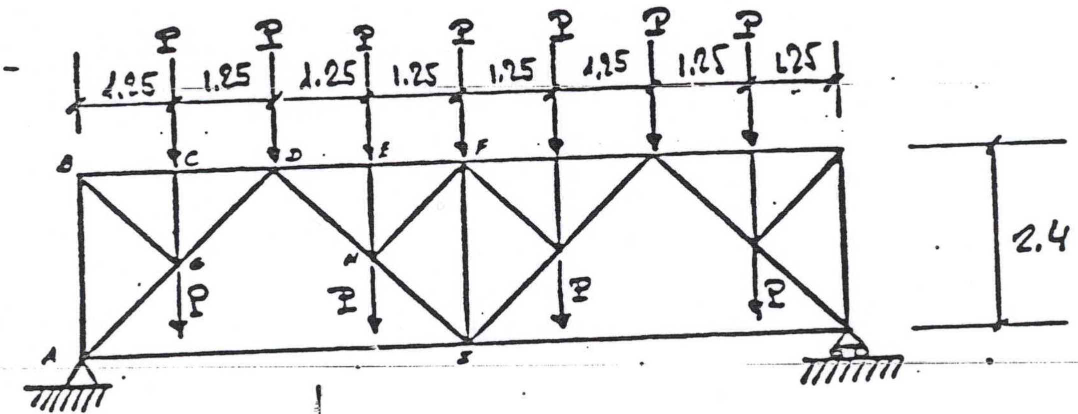
PRUEBAS DE EVALUACIÓN A DISTANCIA

UNIVERSIDAD NACIONAL DE EDUCACION A DISTANCIA

Asignatura: <u>ANALISIS DE ESTRUCTURAS-METODOS NUMERICOS</u>

Problema 1:

Calcular los esfuerzos en las barras de la estructura representada en la figura, mediante el diagrama de Maxwell -Cremona.
P = 1000 Kg. (UTILIZAR LA NOTACIÓN DE BOW).

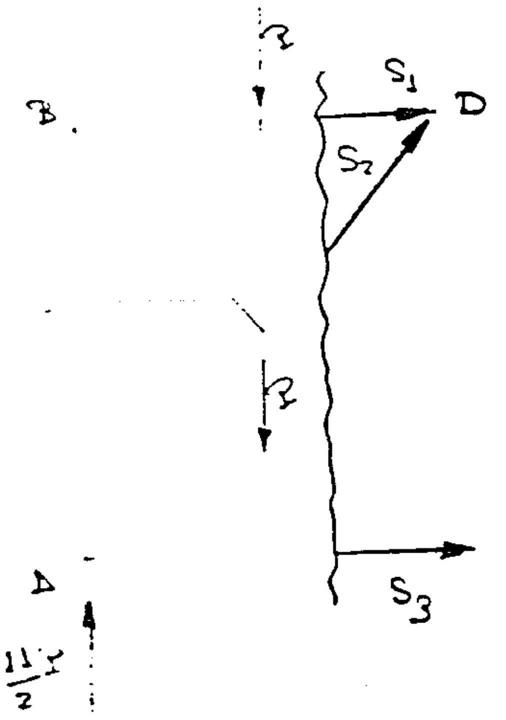


Para poder aplicar el Cremona será necesario aplicar Ritter para calcular el valor de los esfuerzos en las barras del cordón inferior mediante un corte como el que se indica.

Calculamos primeramente las reacciones.

$$R_A = R_{A'} = \frac{11P}{2} = 5500\ kg$$

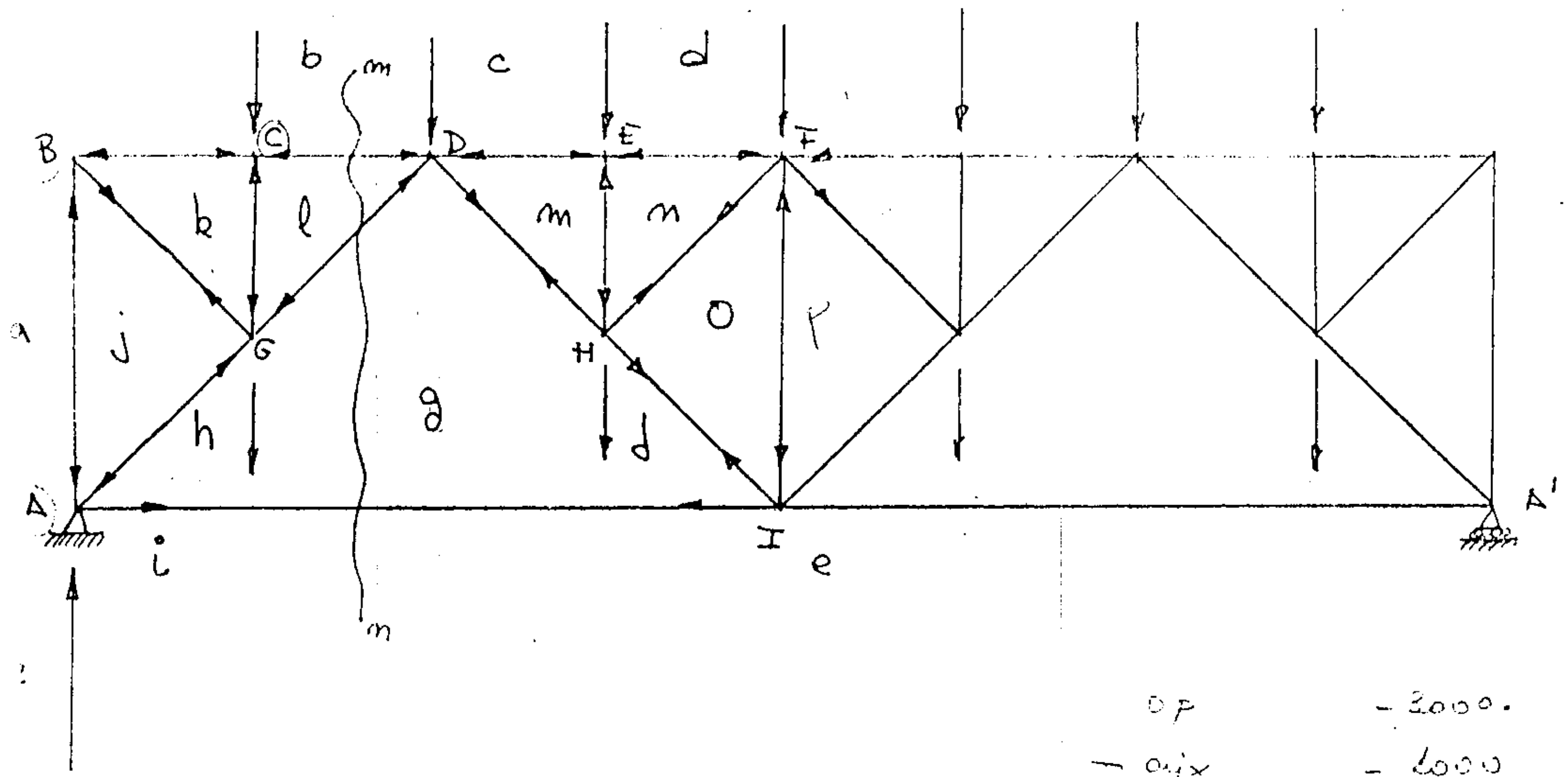
Tomamos momentos respecto a D.

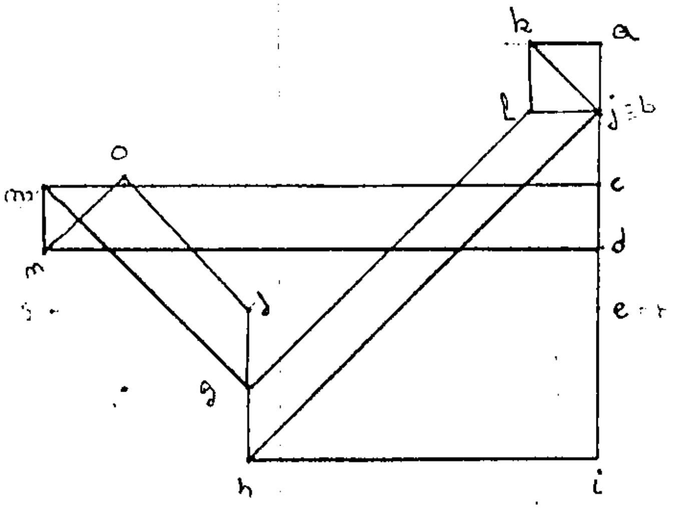


$$-S_3 \cdot h + \frac{11}{2} P \cdot 2a - 2P \cdot a = 0$$

$$S_3 = \frac{9a}{h} P = \frac{9 \times 1.25}{2.4} 1000 = 4687.5\ kg$$ = 46[illegible]

Por tanto podemos sustituir la barra AI por su esfuerzo S_3 y considerarla como una fuerza exterior, por lo que podemos obtener el diagrama de Maxwell con las regiones que se indican en la figura, donde por simetría sólo se ha considerado la mitad de la estructura.





op	– 2000.	FI – x
– ajx	– 2000	AB — x
– ab	– 1100	BC / +
bj	+ 1500	BG / +
– jh	– 6500	AG / +
kl	– 1000	CG / –
bc	– 1100	CD / +
lg	– 5000	GD / +
cm	– 7300	DE / +
ng	+ 3600	DH / +
dn	– 7300	[illegible] / +
mn	– 1000	EH / +
no	+ 1450	[illegible] / +
op	+ 2200	[illegible] +

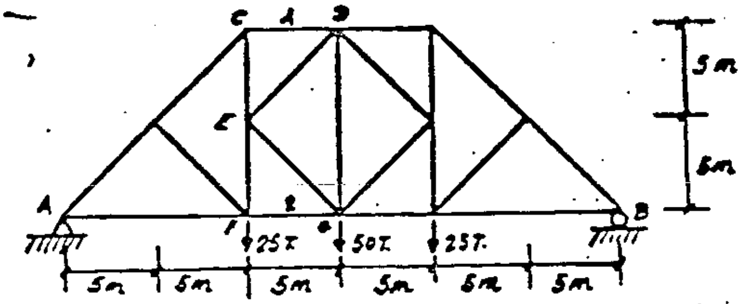
EJERCICIO 2

UNIVERSIDAD NACIONAL DE EDUCACION A DISTANCIA

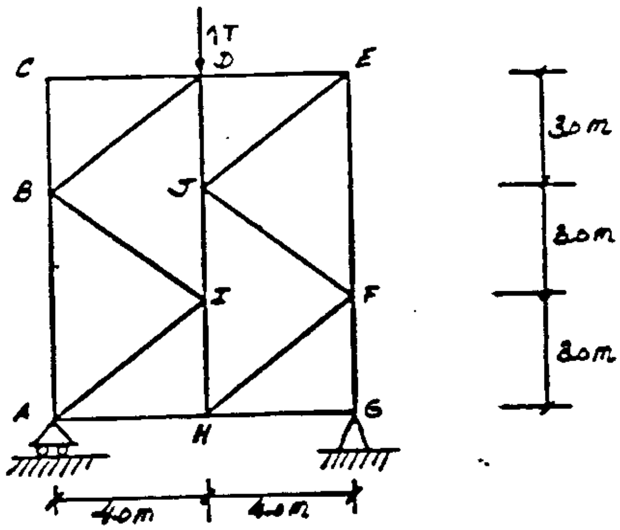
Asignatura: ANALISIS DE ESTRUCTURAS-METODOS NUMERICOS

Problema 2 :

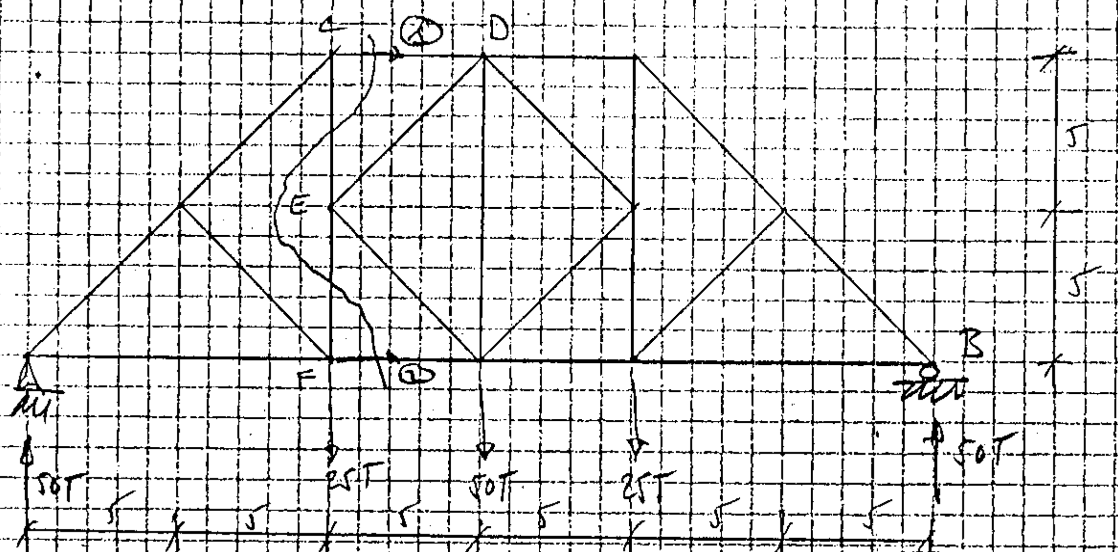
a) Calcular los esfuerzos en las barras 1 y 2 de la estructura representada en la figura aplicando el método de RITTER.



b) Calcular los esfuerzos en todas las barras de la estructura representada en la figura.



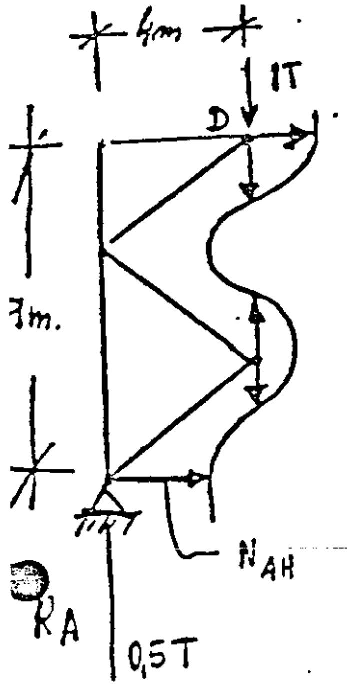
(1)



Mom. resp. C ⇒ $S_2 \times 10 = 50 \times 10$ ⇒ $S_2 = 50$

Mom. resp. F ⇒ $S_1 \times 10 = -50 \times 10$ ⇒ $S_1 = -50$ Compr.

Aplicaremos el método de Ritter para calcular el esfuerzo en la barra AH, y así poder resolver la estructura, por ejemplo, por el método de los nudos.

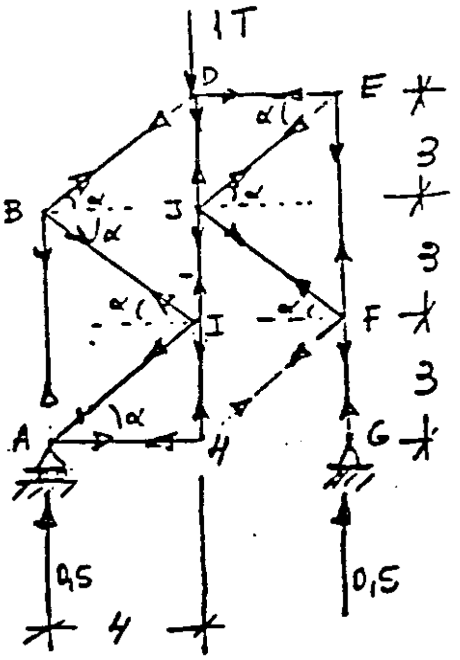


Tomando momentos con respecto a D.

$$R_A \cdot 4 - N_{AH} \cdot 9 = 0 \Rightarrow 500 \cdot 4 = N_{AH} \cdot 9$$

$$N_{AH} = \frac{2000}{9} = 222,2 \text{ Kg.} \checkmark$$

Resolución de la estructura por el método de los nudos:



Nudo A

$$\begin{cases} \operatorname{sen}\alpha = \frac{3}{\sqrt{4^2+3^2}} = \frac{3}{5} \\ \cos\alpha = \frac{4}{5} \end{cases}$$

$$\left.\begin{aligned} \Sigma F_H = 0 &\Rightarrow N_{AB} + N_{AI} \cdot \tfrac{3}{5} + 0,5 = 0 \\ \Sigma F_V = 0 &\Rightarrow N_{AH} + N_{AI} \cdot \tfrac{4}{5} = 0 \end{aligned}\right\}$$

$$N_{AB} + \frac{3}{5} N_{AI} + 0,5 = 0 \quad \checkmark$$

$$0,222 + \frac{4}{5} N_{AI} = 0 \Rightarrow N_{AI} = -\frac{5 \cdot 0,222}{4} = -0,27$$

$$N_{AB} = -0,5 + \frac{3}{5} 0,277 = -0,333 \text{ T.} \checkmark$$

Nudo B

$$\left.\begin{aligned} \Sigma F_V = 0 &\Rightarrow -N_{AB} - N_{BI} \tfrac{3}{5} + N_{BD} \tfrac{3}{5} = 0 \\ \Sigma F_H = 0 &\Rightarrow N_{BI} \tfrac{4}{5} + N_{BD} \tfrac{4}{5} = 0 \end{aligned}\right\}$$

$$0,333 - \frac{3}{5} N_{BI} + \frac{3}{5} N_{BD} = 0$$

$$\frac{4}{5} N_{BI} + \frac{4}{5} N_{BD} = 0 \Rightarrow N_{BI} = -N_{B}$$

$$0.333 + \frac{3}{5} N_{BD} + \frac{3}{5} N_{BD} = 0 \Rightarrow N_{BD} = -0,333 \frac{5}{6} = -0,277 \text{ T} \checkmark$$

Nudo H

$$\Sigma F_V = 0 \Rightarrow N_{HI} + \frac{3}{5} N_{HF} = 0$$

$$\Sigma F_H = 0 \Rightarrow -N_{AH} + \frac{4}{5} N_{HF} = 0$$

$$-0{,}222 + \frac{4}{5} N_{HF} = 0 \Rightarrow N_{HF} = \frac{5 \cdot 0{,}222}{4} = \boxed{0{,}277\,T} \checkmark$$

$$N_{HI} = -\frac{3}{5}\, 0{,}277 = \boxed{-0{,}166\,T} \checkmark$$

Nudo I

$$\Sigma F_V = 0 \Rightarrow -N_{HI} + N_{IJ} - \frac{3}{5} N_{AI} + \frac{3}{5} N_{BI} = 0 \Rightarrow +0{,}166 + N_{IJ} + \frac{3}{5}\, 0{,}277 + \frac{3}{5}\, 0{,}2 \ldots = 0$$

$$N_{IJ} = -\frac{6}{5}\, 0{,}277 - 0{,}166 = \boxed{-0{,}498\,T} \checkmark$$

Nudo D

$$F_V = 0 \Rightarrow -1 - N_{DJ} - \frac{3}{5} N_{BD} = 0$$

$$\Sigma F_H = 0 \Rightarrow -\frac{4}{5} N_{BD} + N_{DE} = 0$$

$$+\frac{4}{5}\, 0{,}277 + N_{DE} = 0 \Rightarrow N_{DE} = \boxed{-0{,}221} \checkmark$$

$$-1 - N_{DJ} + \frac{3}{5}\, 0{,}277 = 0 \Rightarrow N_{DJ} = \frac{3}{5}\, 0{,}277 - 1 = \boxed{-0{,}834\,T} \checkmark$$

Nudo E

$$\Sigma F_V = 0 \Rightarrow -N_{JE}\frac{3}{5} - N_{EF} = 0 \Rightarrow$$

$$\Sigma F_H = 0 \Rightarrow -N_{DE} - N_{JE}\frac{4}{5} = 0 \Rightarrow N_{JE} = +\frac{5}{4}\, 0{,}221 = \boxed{0{,}276\,T} \checkmark$$

$$N_{EF} = -\frac{3}{5} N_{JE} = -\frac{3}{5} \times 0{,}276 = \boxed{-0{,}165\,T} \checkmark$$

Nudo G

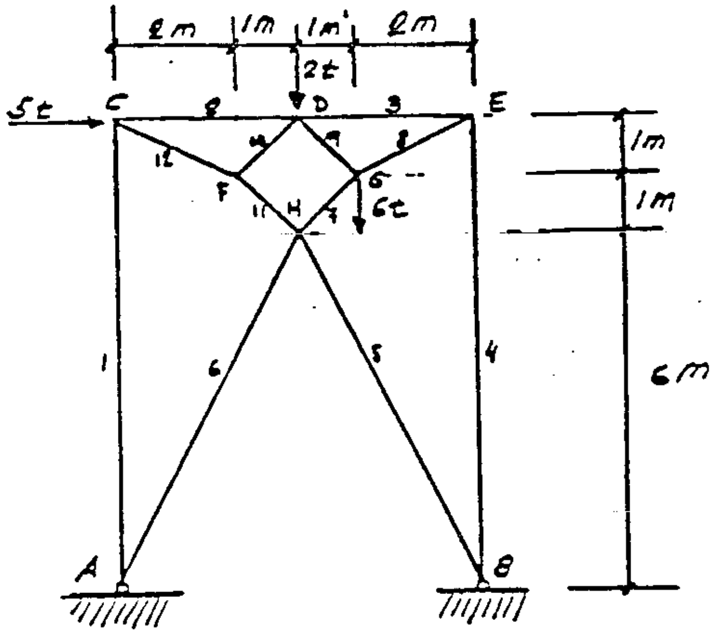
$$0{,}5 + N_{FG} = 0 \qquad N_{FG} = \boxed{-0{,}5\,T} \checkmark$$

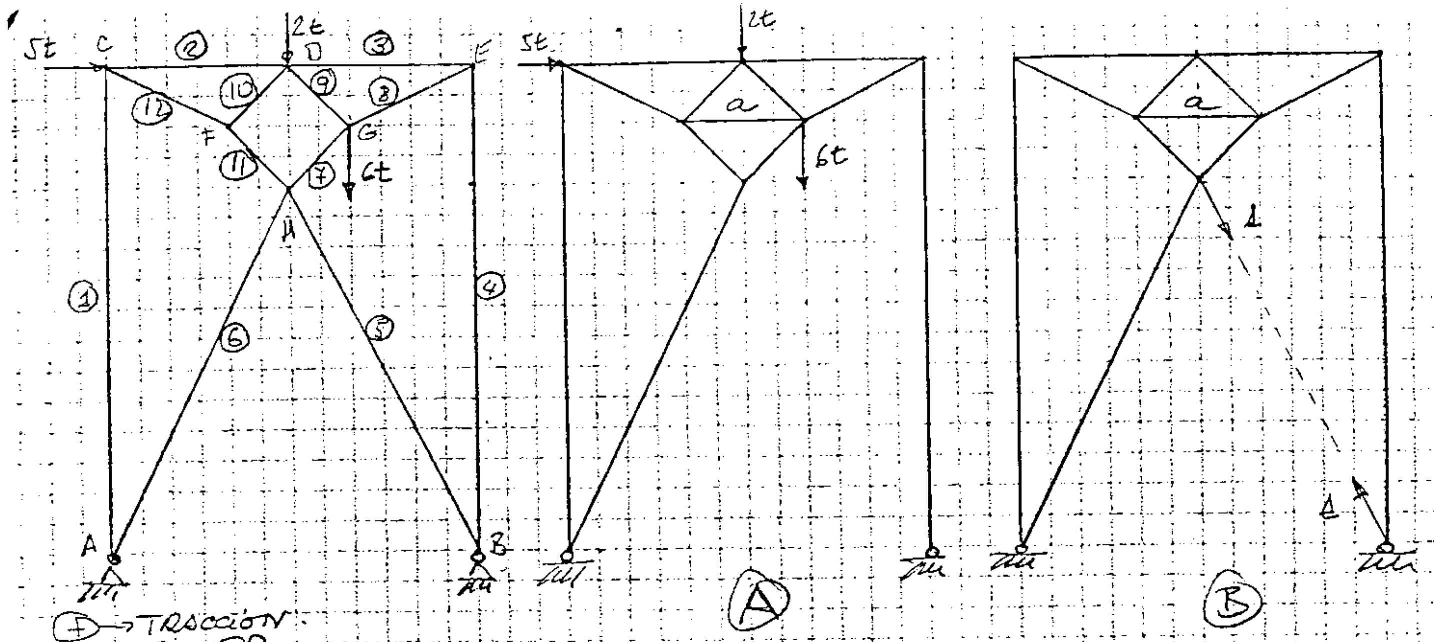
UNIVERSIDAD NACIONAL DE EDUCACION A DISTANCIA

Asignatura: ANALISIS DE ESTRUCTURAS-METODOS NUMERICOS

Problema 3 :

Resolver por el metodo de HENNEBERG la estructura de la figura.





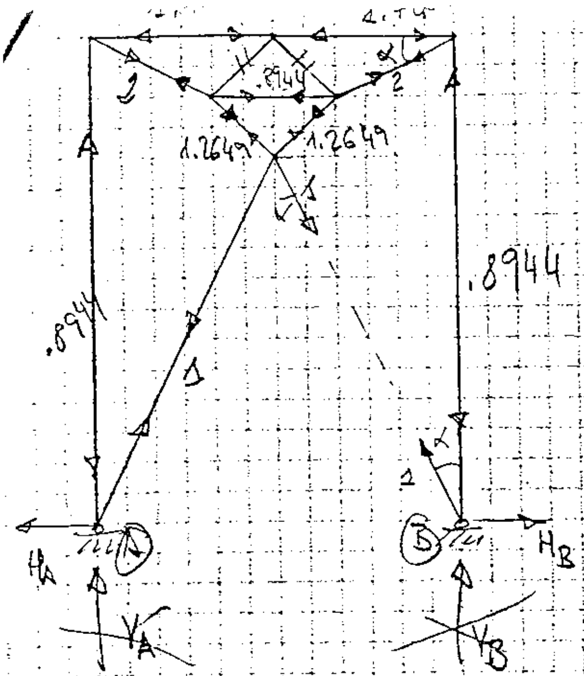
(+) → TRACCIÓN
(−) → COMPR.

BARRA	$S_i^{(A)}$	$S_i^{(B)}$	$\bar{X}\, S_i^{(B)}$	$S_i = S_i^{(A)} + \bar{X}\, S_i^{(B)}$	
1	-6,3333	-.8944	14.-	7,6667	—
2	-17,6666	-1,7889	28.-	10,3351	—
3	-23,3334	-1,7889	28.-	4,6666	—
4	-11,6667	-.8944	14.-	2,3333	—
5	—	—	—	-15,653	
6	11.1803	1.-	-15,653	-4,4727	—
7	10.6066	1.2649	-19,7995	-9,1929	—
8	26,0375	2.-	-31,3060	-5,2185	—
9	2,5928	0.-	0.-	2,5928	
10	-5,4212	0.-	0.-	-5,4212	
11	3,5354	1.2649	-19,7995	-16,2641	—
12	14,1617	2.-	-31,306	-17,1443	—
a	14.-	.8944	-14	0	—

$u_a = 12 + 4 = 16$

$2j = 2 \times 8 = 16$

$$S_a = 0 = S_i^{(A)} + \bar{X}\, S_i^{(B)} = 14 + \bar{X}\; 0{,}8944 \Rightarrow \bar{X} = -\frac{14}{0{,}8944} = -15{,}653$$



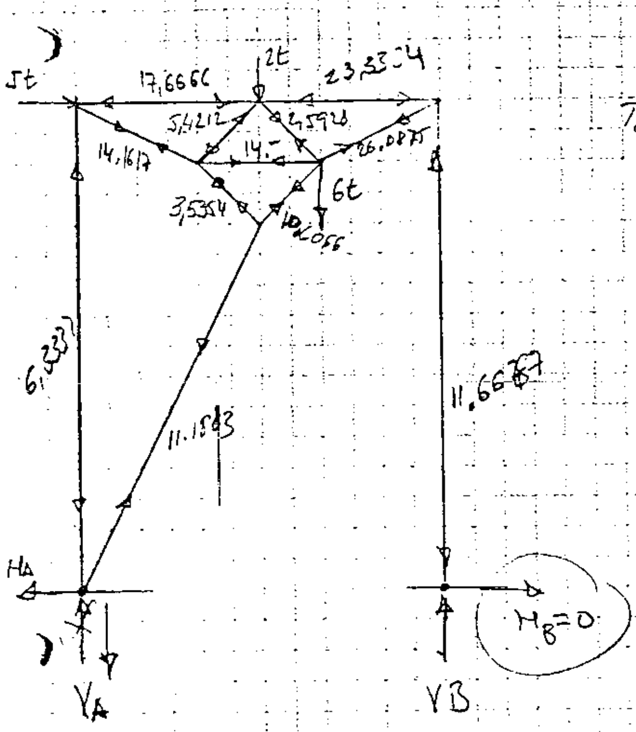
Tomo momentos respecto de A!

$$V_B \times 6 + 1 \cos\alpha \times 6 = 1 \cos\alpha \times 3 + 1 \operatorname{sen}\alpha \times$$

$$V_B = \frac{3 \times \cos\alpha + 6 \times \operatorname{sen}\alpha - 6 \times \cos\alpha}{6} = \underline{0}$$

$$H_B = 1 \times \operatorname{sen}\alpha = \underline{0{,}45}$$

$$H_A = \underline{0{,}45}$$



Tomo mom. resp. de A.

$$V_B \times 6 = 6 \times 4 + 2 \times 3 + 5 \times 8$$

$$V_B = \underline{11{,}6667\ t}$$

$$V_A = 6 + 2 - 11{,}6667 = \underline{-3{,}6667\ t}$$

$$H_A = \underline{5\ t}$$

UNIVERSIDAD NACIONAL DE EDUCACION A DISTANCIA

Asignatura : ANALISIS DE ESTRUCTURAS METODOS NUMERICOS

Problema 4:

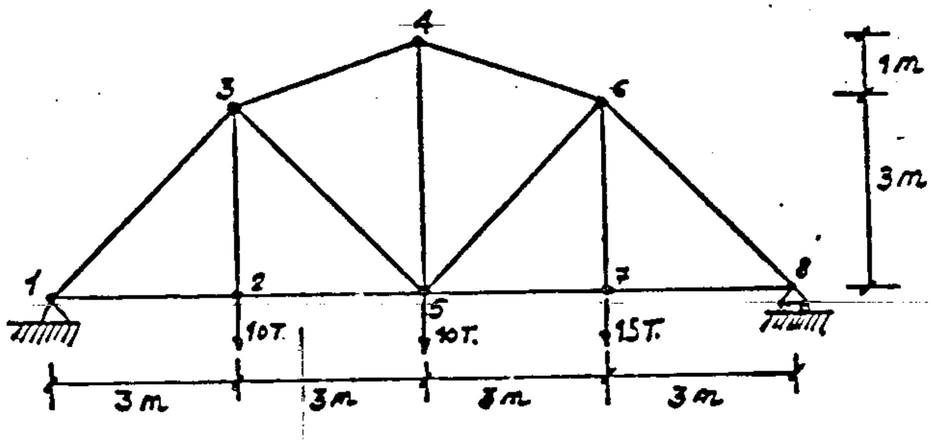
Obtener por el método de Williot la deformada de la estructura

e la figura, sabiendo:

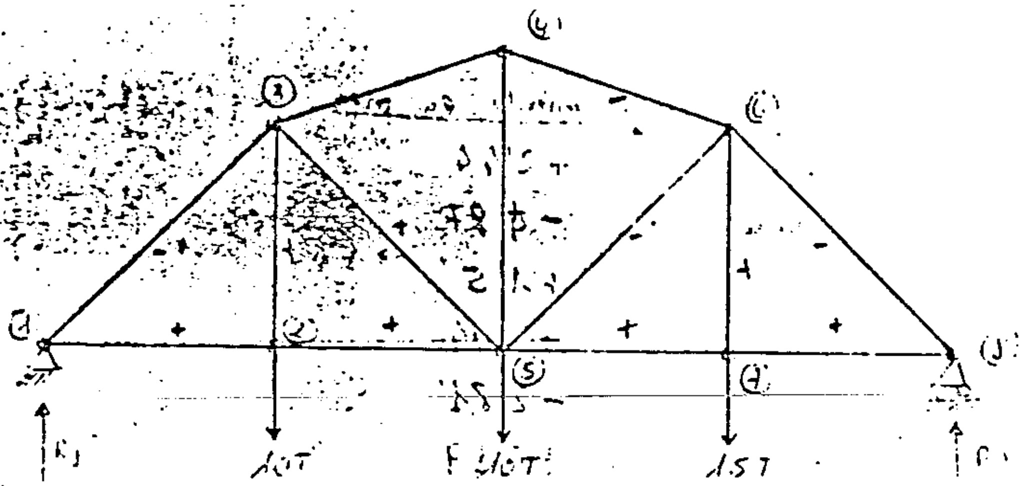
= 2×10^6 Kg/cm^2

=10 cm^2 en todas las barras

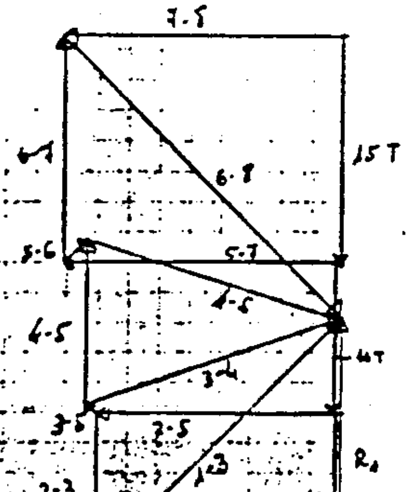
scala : 3 mm = 1 mm de deformación

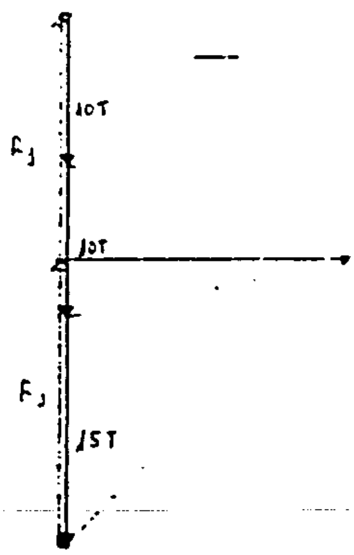


- Calculo de las armaduras
- Calculo de esfuerzo (por Cremona)



Escala 1/5



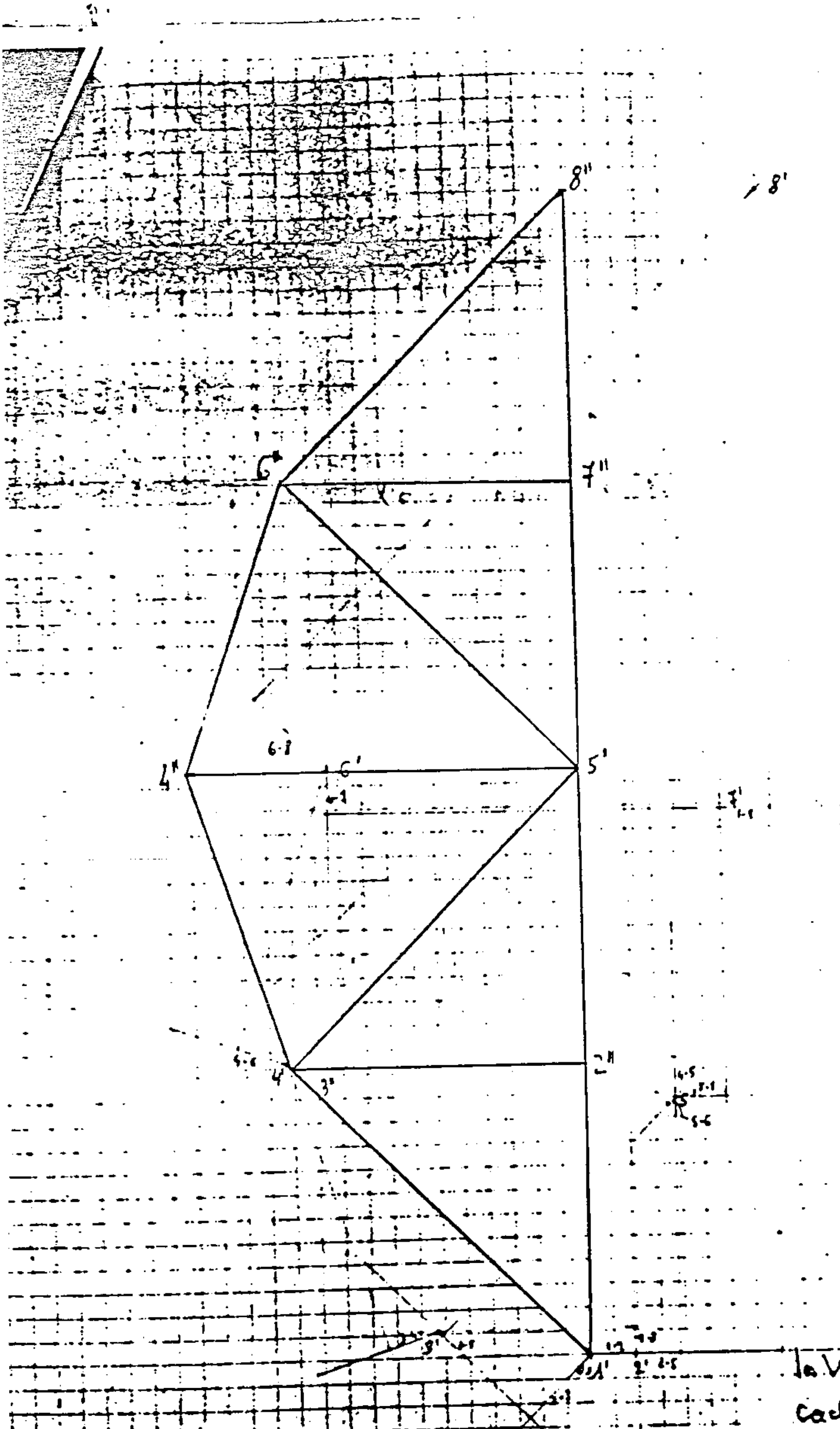


NEGRO - Traccion
ROJO - Compresion

Esfuerzo de cada barra

	Valor en Tn	Tipo	Alargamiento en mm $= \frac{N_i L_i}{E A_i}$
2	16'25	Tracción	+2'44
3	22'98	Compresión	-4'87
3	10	T	+1'5
5	16'25	T	+2'44
4	17'79	C	-2'81
5	0'88	T	+0'19
5	11'25	T	+2'25
6	17'79	C	-2'81
6	2'65	C	-0'56
7	18'75	T	+2'81
7	15	T	+2'25
8	26'52	C	-5'63
8	18'75	T	+2'81
1	16'25 ↑		
8	18'75 ↑		

Alargamiento unitario $\Delta = \frac{L}{E \Omega}$;



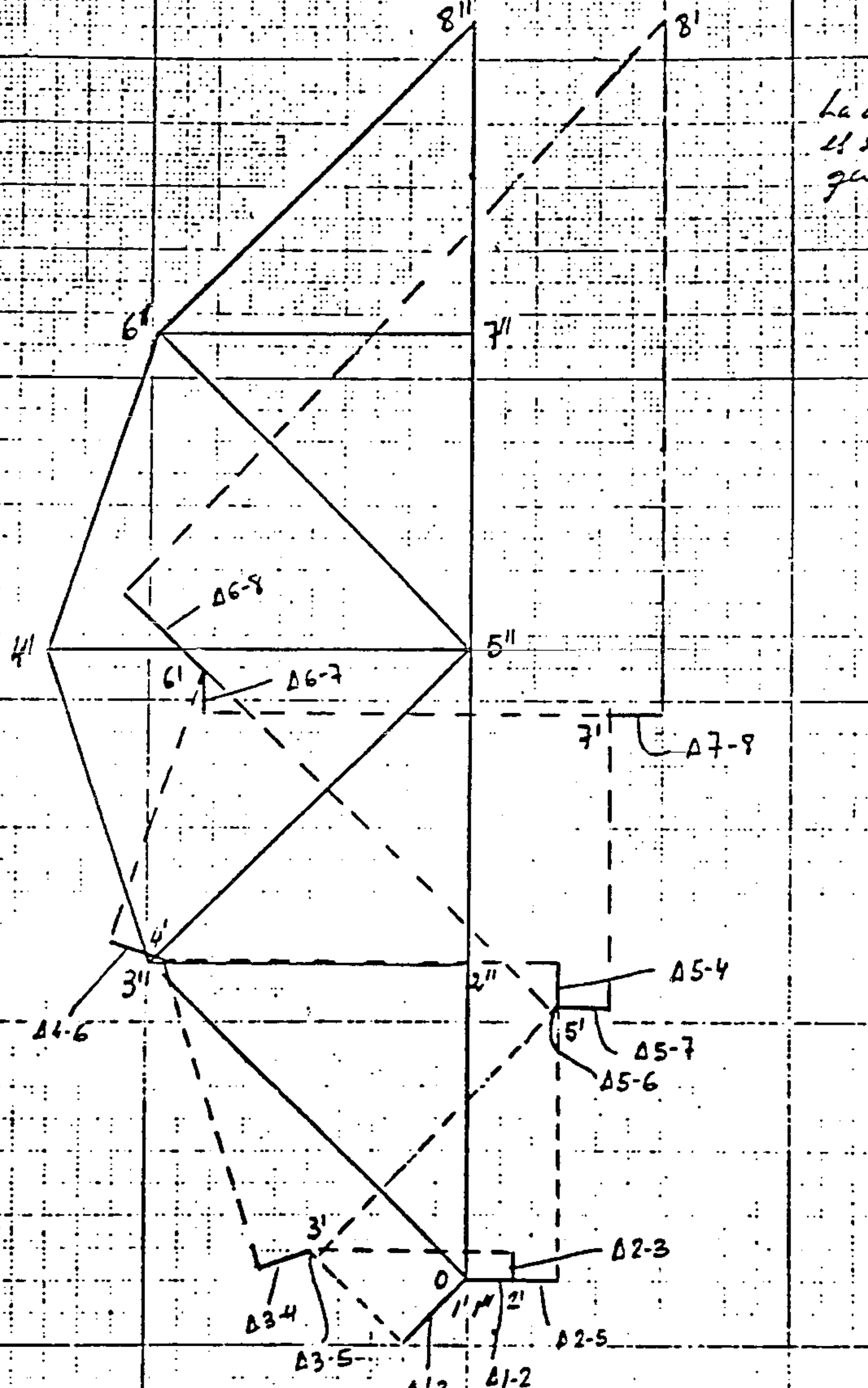
la Vectores corrimientos de cada punto son ('' - ')
ejemplo 8''8'

Valor absoluto del corrimiento de lo nudo ; en cm.

1	2	3	4	5	6	7	8
0	1'64	1'67	1'69	1'9	1'62	2'02	1'09

DIAGRAMA DE WILLIOT

Escala 3cm.cm - 1cm en deformacion



movimientos de los nudos:

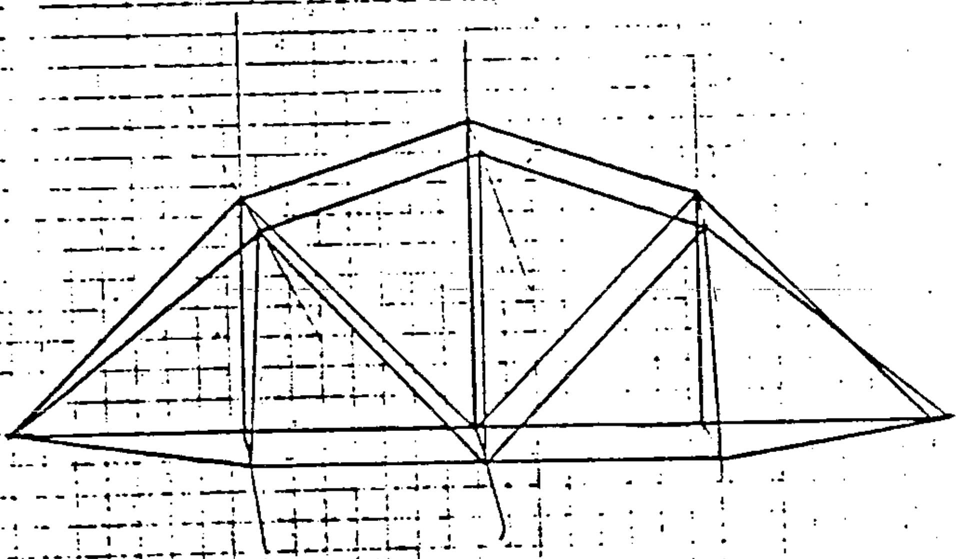
nudo 1 permanece fijo

nudo 2 ⇒ vector $\overrightarrow{2''2'}$ 1.64

nudo 5 ⇒ vector $\overrightarrow{5''5'}$ 1.9

nudo 6 ⇒ vector $\overrightarrow{6''6'}$ 1.67

Deformada- (en rojo)



UNIVERSIDAD NACIONAL DE EDUCACION A DISTANCIA

Asignatura: ANALISIS DE ESTRUCTURAS-METODOS ENERGETICOS

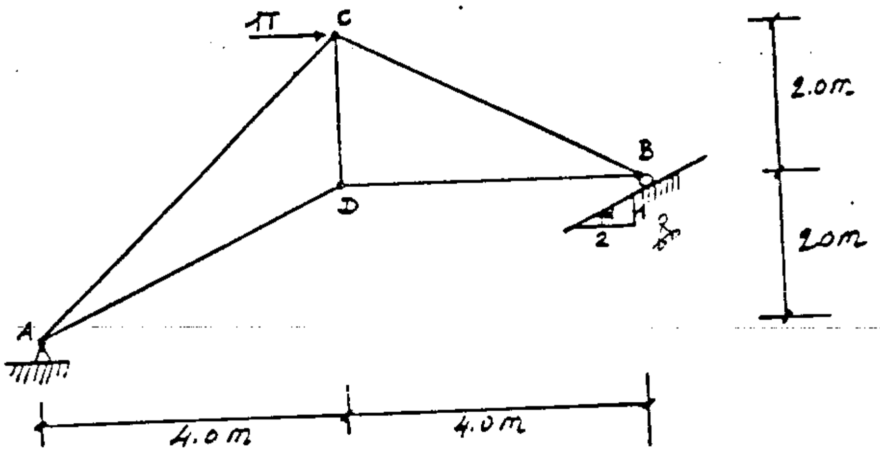
Problema 5:

Obtener el desplazamiento vertical del nudo D de la estructura representada en la figura, por aplicación del método de los trabajos virtuales.

Area de las barras $\overline{AC}, \overline{AD}, \overline{BC}, \overline{BD} = 4.0\ cm^2$

Area de la barra $\overline{CD} = 2.0\ cm^2$

Para todas las barras, $E = 2x10^6\ kg/cm^2$

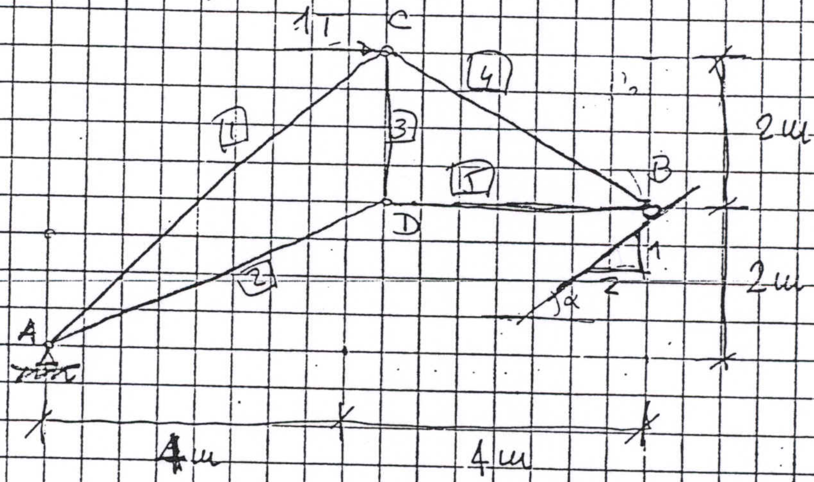


Obtener el desplazamiento vertical del nudo D de la estructura representada en la figura, por aplicación del método de los Trabajos virtuales.

Area de las barras $\overline{AC}$, $\overline{CB}$, $\overline{DB}$, $\overline{AD}$ = 4 cm²

Area de la barra CD = 2 cm²

$E = 2 \times 10^6$ Kg/cm² para todas las barras.



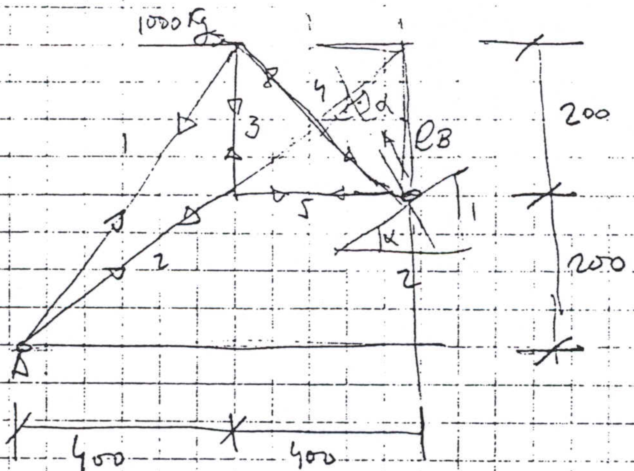
BARRA	N_i^V	N_i^R	L_i/A		$\frac{N_i^V N_i^R L_i}{A}$
1	-1,257	157,159	565,7	/4	-27 938,348
2	0,752	745,342	447,2	/4	62,663,585
3	1,336	333,327	200	/2	44 532,87
4	-0,994	-993,789	447,2	/4	110438,977
5	0,672	666,654	400	/4	44 799,149

$\Sigma = 234\,495,85$

$$\delta = \sum_{i=1}^{5} N_i^V \Delta l_i^R = \sum_{i=1}^{5} N_i^V N_i^R \frac{L_i}{E A_i} = 0,117 \text{ cm}$$

(1)

IITEC



$D = \sqrt{4^2 + 8^2} - 2\,\text{sen}\,\alpha = 8{,}05\ \text{m}$

$\Sigma M_A = 0 \quad 1000 \times 4 = R_B \times 8{,}05$

$R_B = 496{,}894$

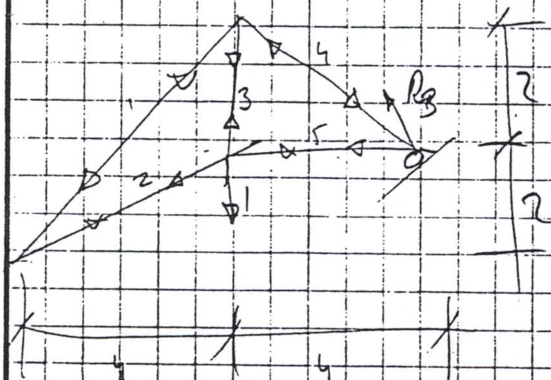
$N_4 = \frac{R_B}{\text{sen}/\cos} = 993{,}789$

$N_5 = 496{,}894 \times \text{sen}\,\alpha - N_4 \cos\alpha = -666{,}654$

$N_2 = \frac{666{,}654}{\cos\alpha} = 745{,}342$

$N_3 = N_2\,\text{sen}\,\alpha = 333{,}327$

$N_1 = \frac{993{,}789 \cos\alpha - 1000}{\cos 45} = -157{,}159$



$R_B = \frac{1 \times 4}{8{,}05} = 0{,}497$

$N_4 = \frac{R_B}{\text{tag}\,\alpha} = 0{,}994$

$\vec{N_5} = 0{,}497\,\text{sen}\,\alpha - 0{,}994 \cos\alpha = -0{,}672$

$N_2 = \frac{0{,}672}{\cos\alpha} = 0{,}752$

$N_3 = 1 + 0{,}752\,\text{sen}\,\alpha = 1{,}336$

$N_1 = \frac{0{,}994 \times \cos\alpha}{\cos 45} = 1{,}257$

UNIVERSIDAD NACIONAL DE EDUCACION A DISTANCIA

Asignatura : ANALISIS DE ESTRUCTURAS - METODOS NUMERICOS

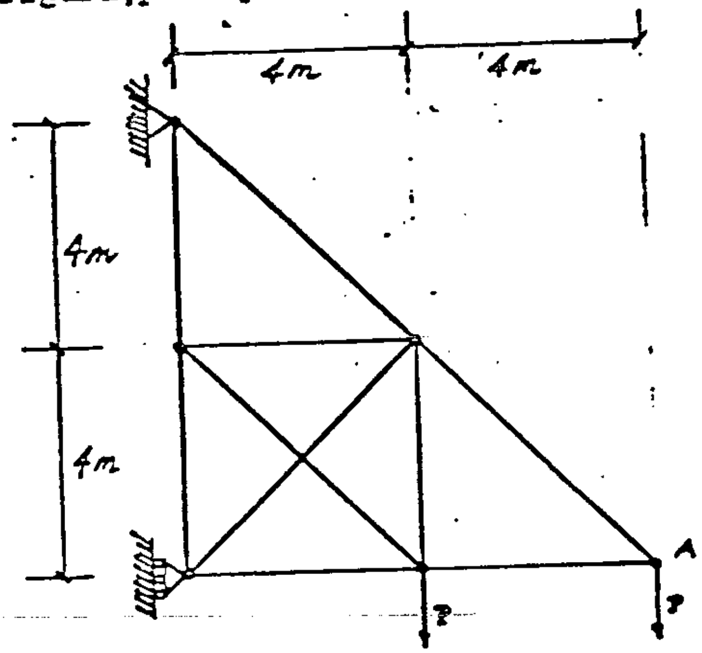
Problema : 1

Calcular el desplazamiento vertical del nudo A ,de la es-
uctura representada en la figura,por aplicación del principio de
s Trabajos Virtuales.

TOS: $P = 2$ T

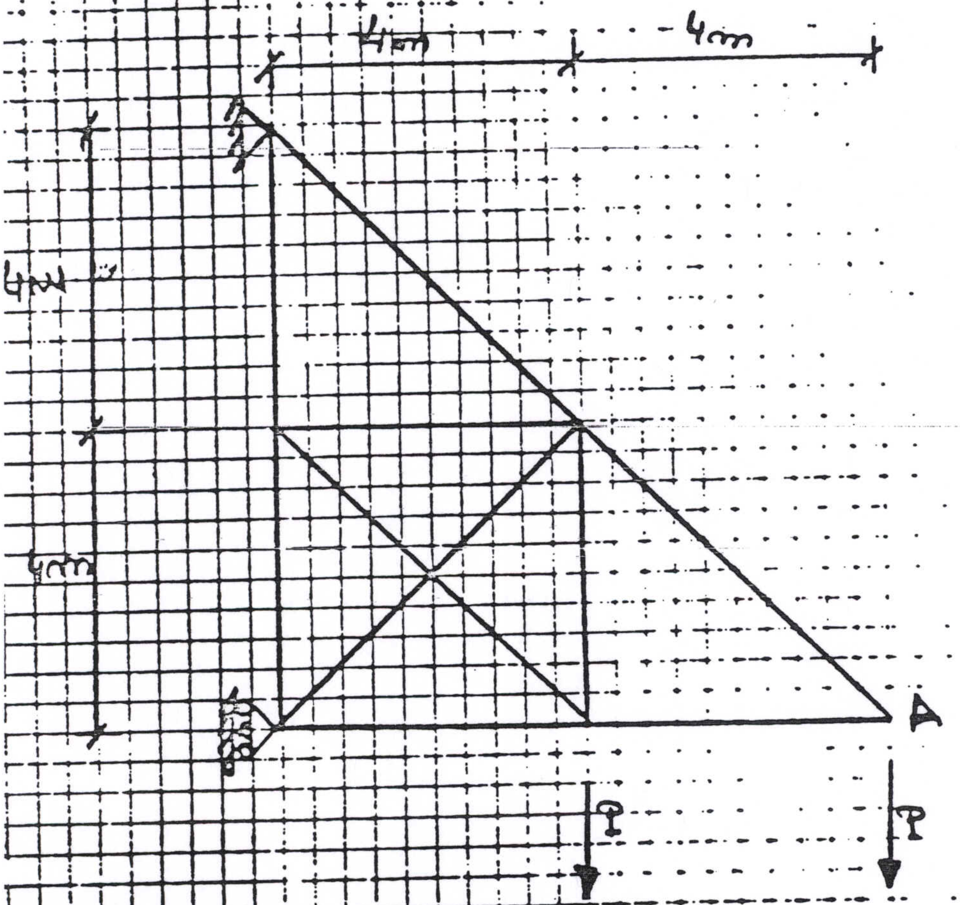
$E = 2 \times 10^6$ Kg/cm^2

$A = 2$ cm^2



FINAL JUNIO 82
1º Parcial.

Para la estructura de la figura se pide: calcular el desplazamiento vertical y horizontal del nudo A por aplicación del Principio de los trabajos virtuales.



P = 2 Ton.

$E = 2 \times 10^6$ kg/cm^2

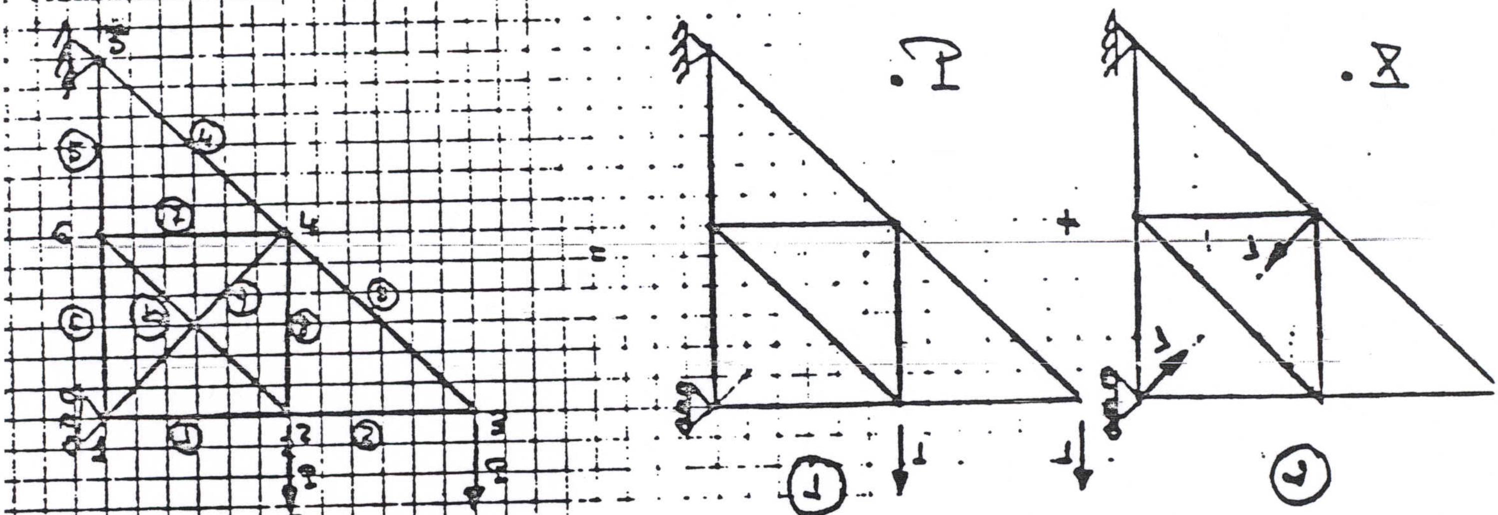
$A = 2$ cm^2

Tiempo: 2 horas.

$1^h\ 45'$

$b = 10 \; ; \; m = 6 \Rightarrow 2m - 3 = 9 < 10$

El sistema es hiperestático interno de grado 1. Tomamos como incógnita hiperestática la barra (9)



Aplicamos el P.T.V.

* Sistema compatible: real.
* Sistema en equilibrio: (2)

$$1 \cdot \Delta_{10} - 1 \cdot \Delta_{10} = \sum_{i=1}^{10} N_i^{(2)} \delta_{REAL}$$

$$\delta_{REAL} = \frac{1}{EA} \left[P \cdot N_i^{(1)} l_i + X N_i^{(2)} l_i \right]$$

$$0 = P \sum N_i^{(1)} N_i^{(2)} l_i + X \sum N_i^{(2)2} l_i$$

BARRA	$N_i^{①}$	$N_i^{②}$	$N_i^{①} \cdot N_i^{②} l_i$	$N_i^{②\,2}$	$N_i^{②\,2} l_i$	l_i
1	$-3/2$	$-\sqrt{2}/2$	$300\sqrt{2}$	$1/2$	200	400
2	-1	0	0	0	0	400
3	$\sqrt{2}$	0	0	0	0	$400\sqrt{2}$
4	$3\sqrt{2}/2$	0	0	0	0	$400\sqrt{2}$
5	$1/2$	0	0	0	0	400
6	0	$\sqrt{2}/2$	0	$1/2$	200	400
7	$-1/2$	$-\sqrt{2}/2$	$100\sqrt{2}$	$1/2$	200	400
8	$1/2$	$-\sqrt{2}/2$	$-100\sqrt{2}$	$1/2$	200	400
9	0	1	0	1	$400\sqrt{2}$	$400\sqrt{2}$
10	$\sqrt{2}/2$	1	400	1	$400\sqrt{2}$	$400\sqrt{2}$

$$\sum N_i^{②\,2} l_i = 800(1+\sqrt{2})$$

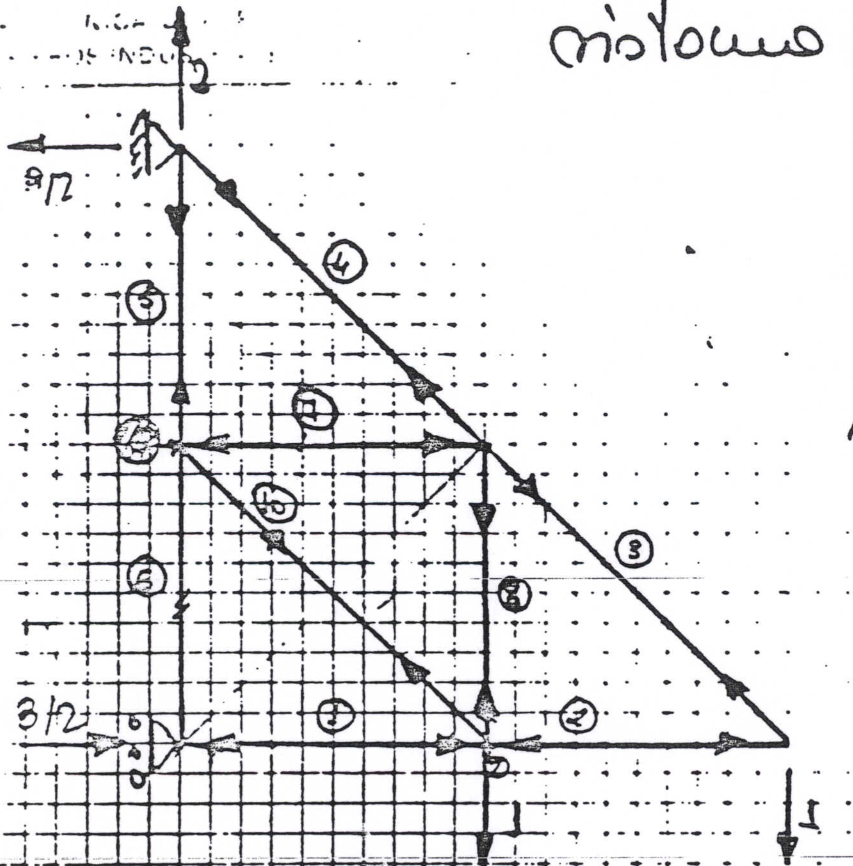
$$\sum N_i^{①} N_i^{②} l_i = 300\sqrt{2} + 400$$

$$0 = P(400 + 300\sqrt{2}) + 800X(1+\sqrt{2}) \Rightarrow X = -\frac{P(400+300\sqrt{2})}{800(1+\sqrt{2})}$$

$X = -853{,}5534$ kg

COMPRIMIDA

تمرين (1)



$N_1 = -3/2$

$N_2 = -1$

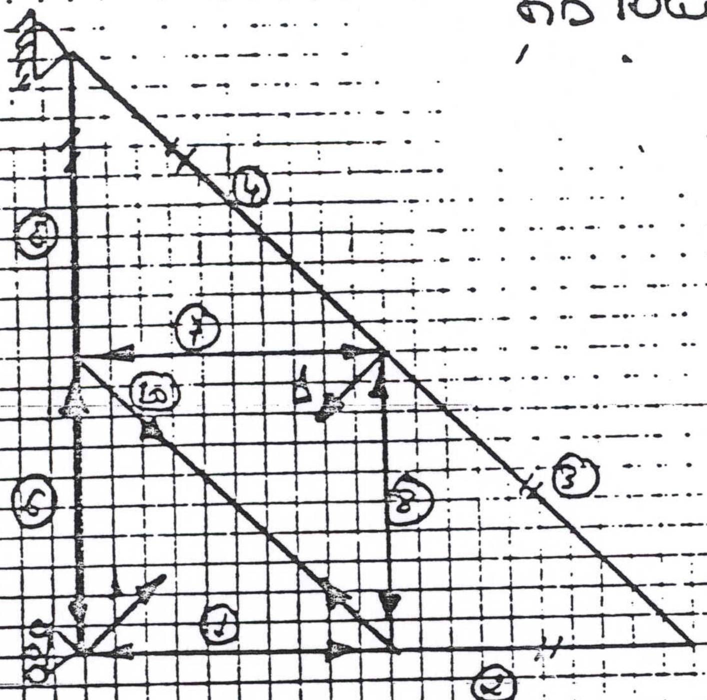
$N_3 = \sqrt{2}$

(2)

$$F_{10} \cos 45 = F_1 - F_2 = \frac{3}{2} - 1 = \frac{1}{2}$$

$$\boxed{F_{10} = \frac{1}{2} \cdot \frac{2}{\sqrt{2}} = \frac{\sqrt{2}}{2}}$$

Sistema (2)

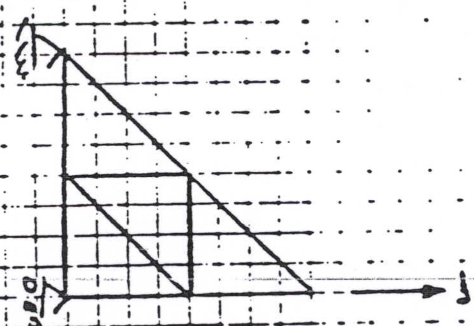


(5)

Calculada la incógnita hiperestática, calculamos el desplazamiento horizontal y vertical del nudo A.

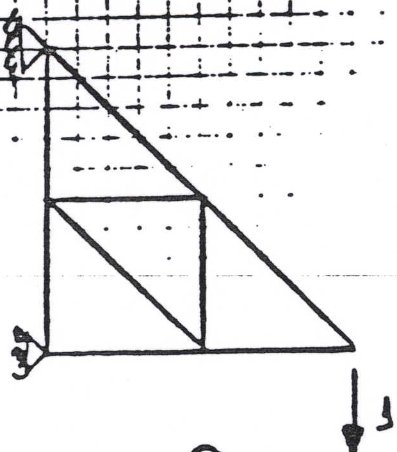
Para ello volvemos a aplicar el P.T.V.

- Sol. compatible: Real.
- Sol. equilibrio (8)



$$u_{HA}\cdot 1 = \sum N_i^{(8)} \delta_{REAL}$$

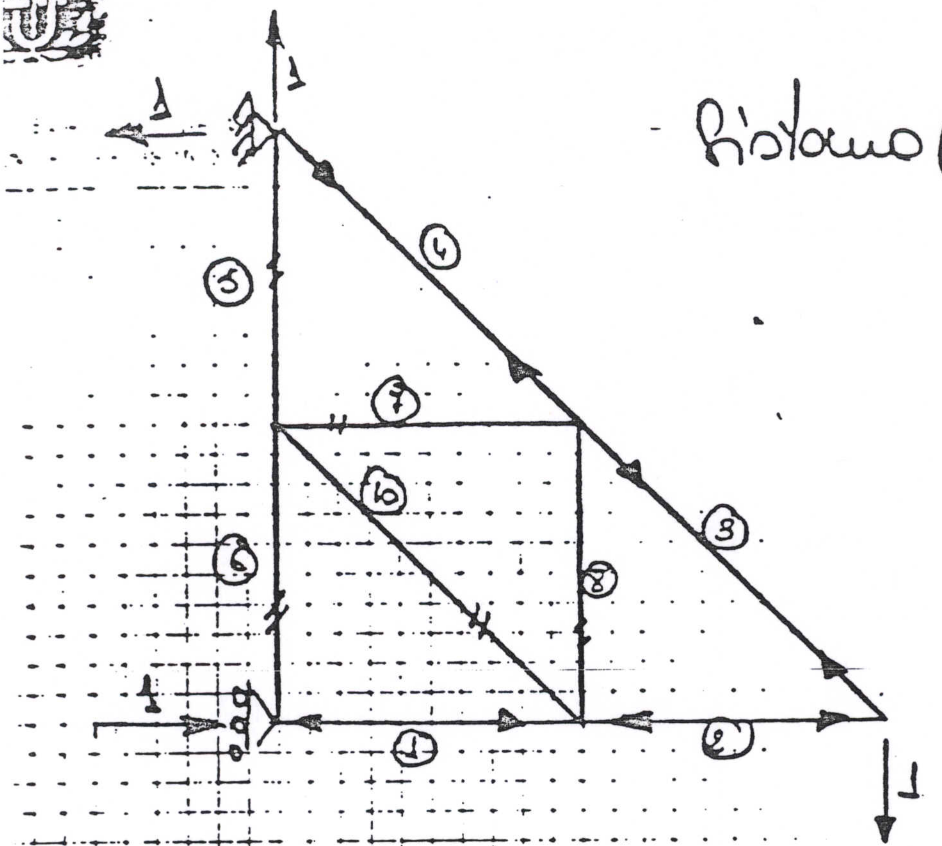
$$u_{HA} = \frac{1}{EA}\left[2\sum N_i^{(8)} N_i^{(1)} l_i + X\sum N_i^{(8)} N_i^{(7)} l_i\right]$$



$$u_{VA}\cdot 1 = \sum N_i^{(8)} \delta_{REAL}$$

$$u_{VA} = \frac{1}{EA}\left[2\sum N_i^{(8)} N_i^{(1)} l_i + X\sum N_i^{(8)} N_i^{(7)} l_i\right]$$

Sistema (8) (6) (mov. vertical)

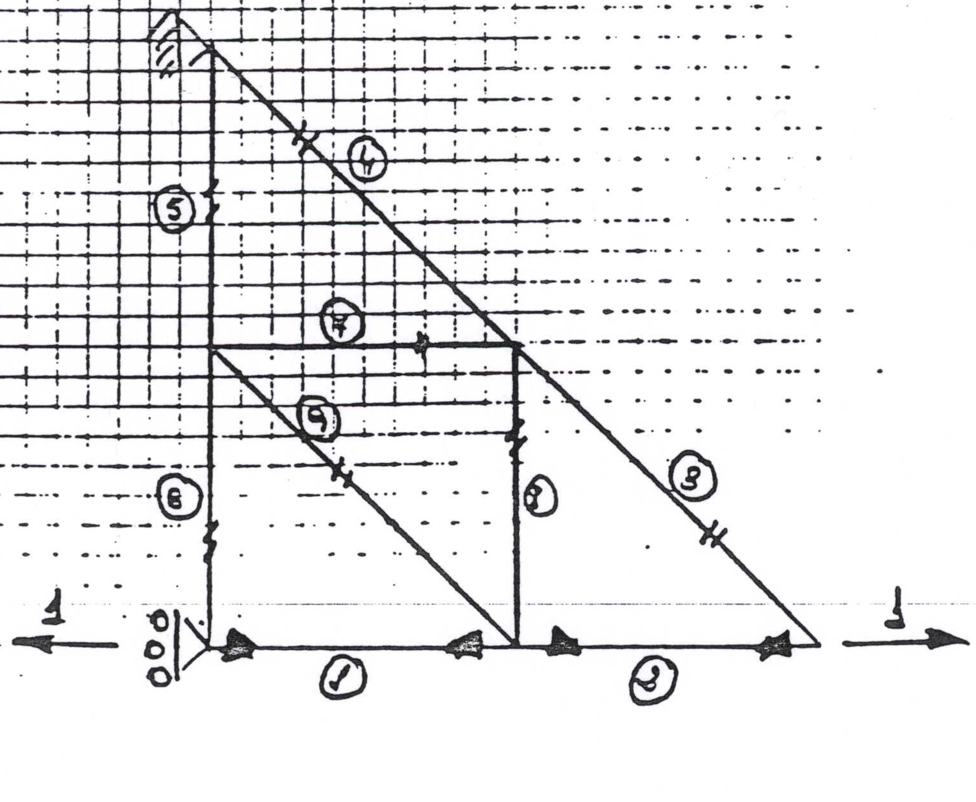


$N_1 = -1$

$N_2 = -1$

$N_3 = N_4 = \sqrt{2}$

Sistema (8) (mov. horizontal)



$N_1 = N_2 = +1$

Doppleronimento

coslirool

(7)

ASTA	$N_i^{(1)}$	$N_i^{(9)}$	$N_i^{(3)}$	$N_i^{(1)} N_i^{(3)} l_i$	$N_i^{(3)} N_i^{(9)} l_i$	l_i
1	$-3/2$	$-\sqrt{2}/2$	-1	600	$200\sqrt{2}$	400
2	-1	0	-1	400	0	400
3	$\sqrt{2}$	0	$\sqrt{2}$	$800\sqrt{2}$	0	$400\sqrt{2}$
4	$\frac{3\sqrt{2}}{2}$	0	$\sqrt{2}$	$1200\sqrt{2}$	0	$400\sqrt{2}$
5	$1/2$	0	0	0	0	400
6	0	$-\sqrt{2}/2$	0	0	0	400
7	$-1/2$	$-\sqrt{2}/2$	0	0	0	400
8	$1/2$	$-\sqrt{2}/2$	0	0	0	400
9	0	1	0	0	0	$400\sqrt{2}$
10	$\sqrt{2}/2$	1	0	0	0	$400\sqrt{2}$

$\hookrightarrow 2000\sqrt{2}+1000$ $\hookrightarrow 200\sqrt{2}$

$$= \frac{1}{4\cdot 10^3}\left[P\left(2000\sqrt{2}+1000\right)+\frac{3\left(400+300\sqrt{2}\right)}{800\left(1+\sqrt{2}\right)}\cdot 200\sqrt{2}\right]$$

$u_{VA} = 1.85386$ mm.

(8)

Desplazamiento horizontal

BARRA	$N_i^{(1)}$	$N_i^{(2)}$	$N_i^{(3)}$	$N_i^{(1)} N_i^{(3)} l_i$	$N_i^{(3)} N_i^{(2)} l_i$	l_i
1	$-3/2$	$-\sqrt{2}/2$	$+1$	600	$200\sqrt{2}$	400
2	-1	0	$+1$	400	0	400
3	$\sqrt{2}$	0	0	0	0	$400\sqrt{2}$
4	$3\sqrt{2}/2$	0	0	0	0	$400\sqrt{2}$
5	$1/2$	0	0	0	0	400
6	0	$-\sqrt{2}/2$	0	0	0	400
7	$-1/2$	$-\sqrt{2}/2$	0	0	0	400
8	$1/2$	$-\sqrt{2}/2$	0	0	0	400
9	0	1	0	0	0	$400\sqrt{2}$
10	$\sqrt{2}/2$	1	0	0	0	$400\sqrt{2}$
				1000	$200\sqrt{2}$	

$$u_{HA} = \frac{1}{4\cdot 10^{8}}\left[P\cdot 1000 - \frac{1(400+300\sqrt{2})}{800(1+\sqrt{2})}\cdot 200\sqrt{2}\right]$$

$u_{HA} = -0.439645$ cm.

BARRA	$N^{(2)}$	$N^{(3)}$	$N^{(3)} N^{(2)} l_i$	$N^{(2)2} l_i$
1	$-\sqrt{2}/2$	-1	$200\sqrt{2}$	200
2	0	-1	0	0
3	0	$\sqrt{2}$	0	0
4	0	$\sqrt{2}$	0	0
5	0	0	0	0
6	$-\sqrt{2}/2$	0	0	200
7	$-\sqrt{2}/2$	0	0	200
8	$-\sqrt{2}/2$	0	0	200
9	-1	0	0	$400\sqrt{2}$
10	1	0	0	$400\sqrt{2}$
			$200\sqrt{2}$	$800(1+\sqrt{2})$

$$200\sqrt{2} = -800\,X\,(1+\sqrt{2})$$

$$X = -\frac{200\sqrt{2}}{800(1+\sqrt{2})} = \frac{\sqrt{2}}{4(1+\sqrt{2})} = 0{,}1464$$

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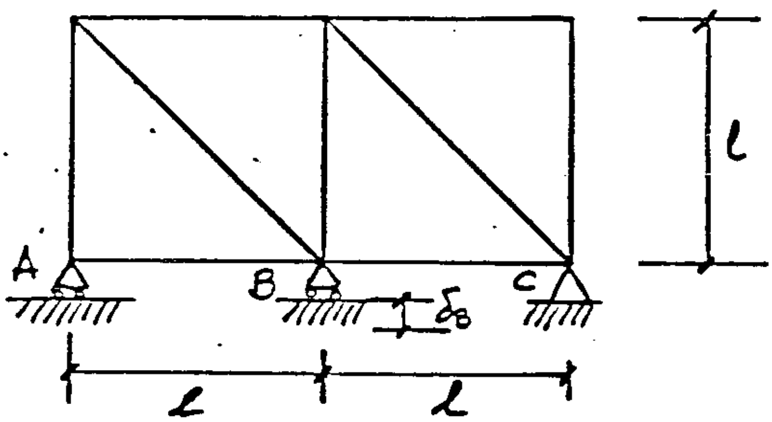
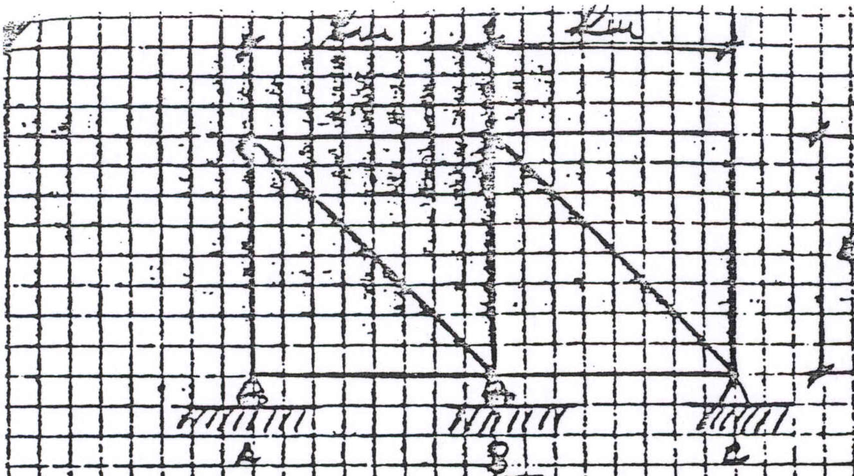
Asignatura: ANALISIS DE ESTRUCTURAS-METODOS NUMERICOS

Problema : 2

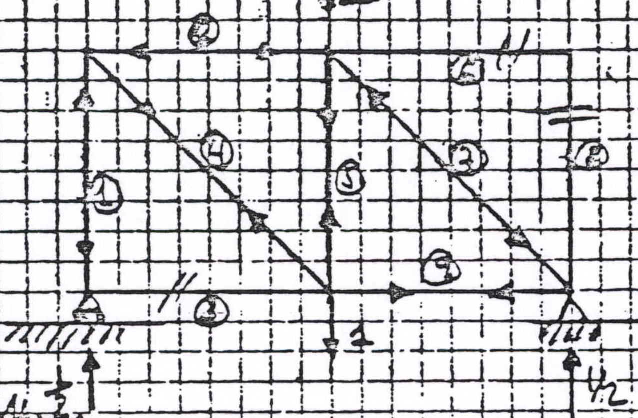
Calcular la reacción " X " en el Punto B de la estructura representada en la figura, cuando se produce un descenso en el apoyo B de 1 cm. (δ_B = 1 cm). Todas las barras tienen el mismo módulo de elasticidad (E) y la misma sección (A).





Barre	F_i	l_i	$l_i F_i^2$
1	$-1/2$	l	$l\,1/4$
2	$-1/2$	l	$l\,1/4$
3	0	l	0
4	$\sqrt{2}/2$	$\sqrt{2}\,l$	$l\,\sqrt{2}/2$
5	$1/2$	l	$l\,1/4$
6	0	l	0
7	$-\sqrt{2}/2$	$\sqrt{2}\,l$	$l\,\sqrt{2}/2$
8	0	l	0
9	$1/2$	l	$l\,1/4$

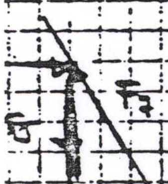
$$\Sigma = l(1+\sqrt{2})$$



…n de los F_i

$F_1 = \frac{1}{2}$; $F_3 = 0$ $\qquad$ $F_6 = F_8 = 0$

$$\Rightarrow \quad F_4 = \frac{1}{2}\,\frac{1}{\sqrt{2}/2} = \frac{\sqrt{2}}{2} \; ; \quad F_2 = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{1}{2}$$



$$\Rightarrow \quad F_7 = \frac{1}{2}\,\frac{1}{\sqrt{2}/2} = \frac{\sqrt{2}}{2} \; ; \quad F_5 = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{1}{2}$$

$$\Rightarrow \quad F_9 = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{1}{2}$$

$$\Delta l_i = \frac{F_i\,X\,l_i}{A_i E}$$

$$\sum\left(F_i X \frac{l_i}{A_i E}\right) F_i = \delta_B = \Delta = \frac{X}{AE}\sum F_i^2 l_i$$

$$X = \frac{\delta\,AE}{\sum F_i^2 l_i} = \frac{\delta\,AE}{l(1+\sqrt{2})} = \frac{0{,}01\cdot E\cdot A}{l(1+\sqrt{2})}$$

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Asignatura: ANALISIS DE ESTRUCTURAS-METODOS NUMERICOS

Problema 3 :

En la estructura de la figura se produce un incremento de temperatura en todas las barras de 40° C.

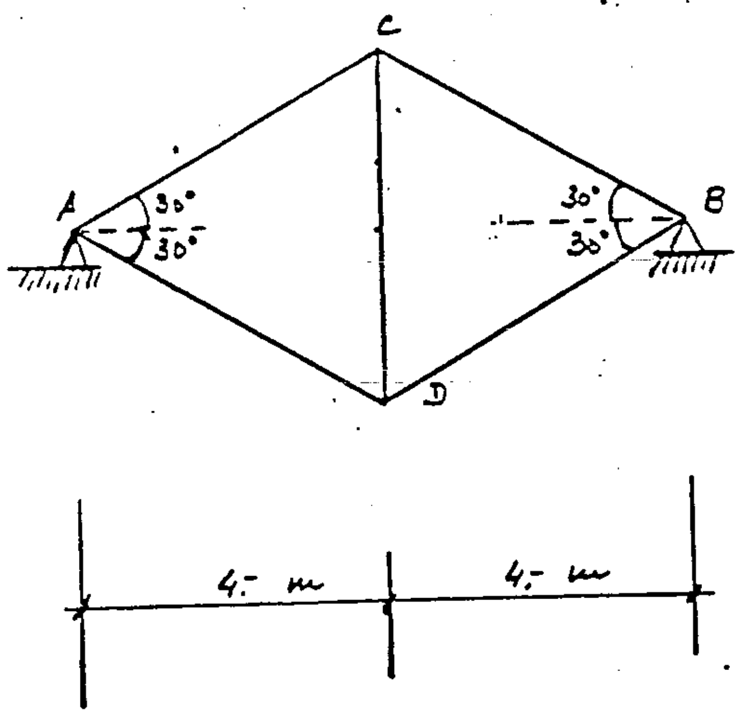
Calcular:

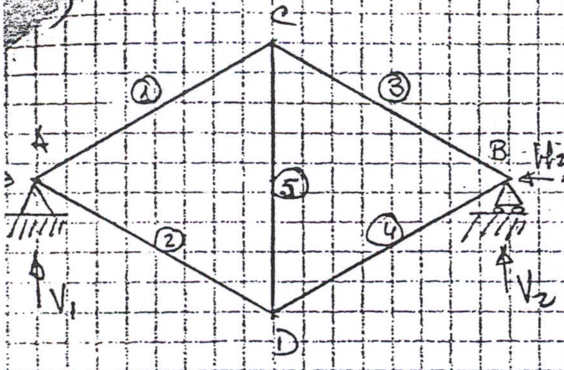
a)los esfuerzos en las barras y reacciones en los apoyos.

b)el desplazamiento vertical del punto C

DATOS: $\alpha = 12 \times 10^{-6}\ °C^{-1}$ || Area igual para todas las barras(A) $A = 2\ cm^2$

$E = 2,1 \times 10^{6}\ Kg/cm^2$





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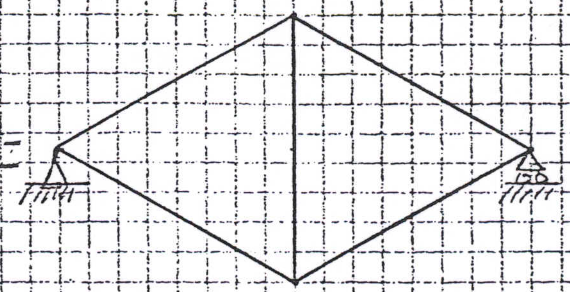
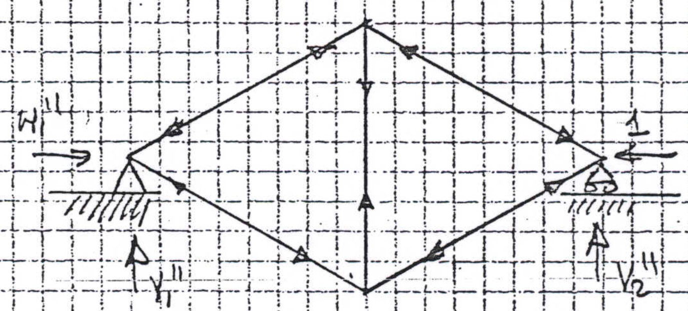


(ΔT) (I)

(ΔT)

+

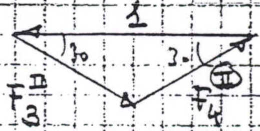


$\times H_2$ (II)

(I) $\Rightarrow$ $F_i^{I} = 0$

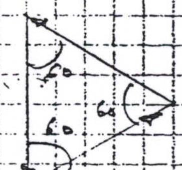
(II) $\Rightarrow$ $H_1'' = 1$

$V_1'' = V_2'' = 0$



$F_3^{II} = F_4^{II} = \frac{1}{2} \frac{1}{\cos 30} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}}$

$= \frac{1}{\sqrt{3}}$



$F_1^{II} = \frac{1}{\sqrt{3}}$

$F_2^{II} = \frac{1}{\sqrt{3}}$

$F_5^{II} = \frac{1}{\sqrt{3}}$

− → COMPRESIÓN

+ → TRACCIÓN

BARRA	l_i	F_i^{I}	F_i^{II}	Δl_{real_i}	$\Delta l_{real_i} \times F_i^{II}$
1	l	0	$-\sqrt{3}/3$	$-\frac{H_2 l}{\sqrt{3} AE} + \alpha \Delta T l$	$\frac{H_2 l}{3AE} - \frac{\sqrt{3}}{3} \alpha \Delta T l$
2	l	0	$-\sqrt{3}/3$	"	"
3	l	0	$-\sqrt{3}/3$	"	"
4	l	0	$-\sqrt{3}/3$	"	"
5	l	0	$\sqrt{3}/3$	$\frac{H_2 l}{\sqrt{3} AE} + \alpha \Delta T l$	$\frac{H_2 l}{3AE} + \frac{\sqrt{3}}{3} \alpha \Delta T l$

$$\Delta l_{real_i} = H_2 \frac{F_i^{II} l_i}{A_i E} + \alpha \Delta T l_i //$$

Desplazamiento horizontal del punto B = 0

$$0 = \sum_{i=1}^{5} \Delta l_{real_i} \times F_i^{II} = 4\frac{H_2 l}{3AE} - 4\frac{\sqrt{3}}{3}\alpha \Delta T l + \frac{H_2 l}{3AE} + \frac{\sqrt{3}}{3}\alpha \Delta T l$$

que son las fuerzas debidas a la carga 1 colocada en 5 horizontalmente.

$$= 5\frac{H_2 l}{3AE} - 3\frac{\sqrt{3}}{3}\alpha \Delta T l = 0$$

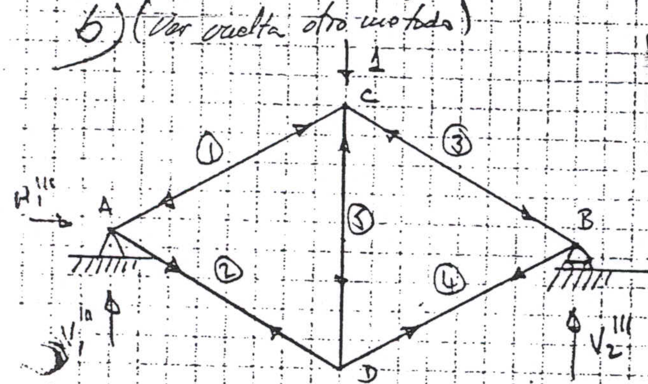
$$\frac{5 H_2 l}{3AE} = \sqrt{3}\,\alpha \Delta T l$$

$$H_2 = \frac{3\sqrt{3}}{5}\alpha \Delta T AE =$$

$$H_2 = \frac{3\sqrt{3}}{5}\, 12 \times 10^{-6} \times 40 \times 2 \times 2{,}1 \times 10^{6} =$$

$$= 2095{,}09 \text{ Kg}$$

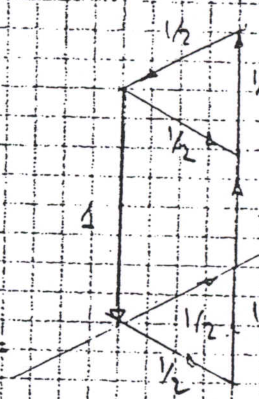
b) (Ver vuelta otro método)



$$l = \frac{4}{\cos 30} = \frac{4}{\sqrt{3}/2} = \frac{8}{\sqrt{3}}$$

$$H_1^{III} = 0$$

$$V_1^{III} = V_2^{III} = \frac{1}{2}$$



BARRA	l_i	F_i^{III}	Δl_{real_i}
1	l	$-1/2$	$-\frac{H_2 l}{\sqrt{3}AE} + \alpha \Delta T l$
2	l	$1/2$	"
3	l	$-1/2$	"
4	l	$1/2$	"
5	l	$-1/2$	$\frac{H_2 l}{\sqrt{3}AE} + \alpha \Delta T l$

$-0{,}0017736\dot{4}$

$$1\delta_c = \sum F_i^{III} \times \Delta l_{real_i} = -\frac{1}{2}\left(-\frac{H_2 l}{\sqrt{3}AE} + \alpha \Delta T l\right) + \frac{1}{2}\left(-\frac{H_2 l}{\sqrt{3}AE} + \alpha \Delta T l\right) - \frac{1}{2}\left(-\frac{H_2 l}{\sqrt{3}AE} + \alpha \Delta T l\right)$$

$$+ \frac{1}{2}\left(-\frac{H_2 l}{\sqrt{3}AE} + \alpha \Delta T l\right) - \frac{1}{2}\left(\frac{H_2 l}{\sqrt{3}AE} + \alpha \Delta T l\right) =$$

$$= -\frac{1}{2}\frac{H_2 l}{\sqrt{3}AE} - \frac{1}{2}\alpha \Delta T l = -\frac{3\sqrt{3}}{5}\alpha \Delta T AE \frac{1}{2}\frac{l}{\sqrt{3}AE} - \frac{1}{2}\alpha \Delta T l = -\frac{4}{5}\alpha \Delta T l =$$

2. Al ser simétrica se calcula el alargamiento de la barra 5 y se divide por 2 obteniendo el despl. pedido.

$$\delta_5 = \frac{1}{2}\left\{\frac{T_5 \cdot l_5}{AE} + \alpha \Delta T \, l_5\right\} = \frac{1}{2}\left\{\frac{210 \times 461{,}88}{2 \times 2{,}1 \times 10^6}\right\} = \frac{0{,}355}{2} = 0{,}17731\,cm \uparrow$$

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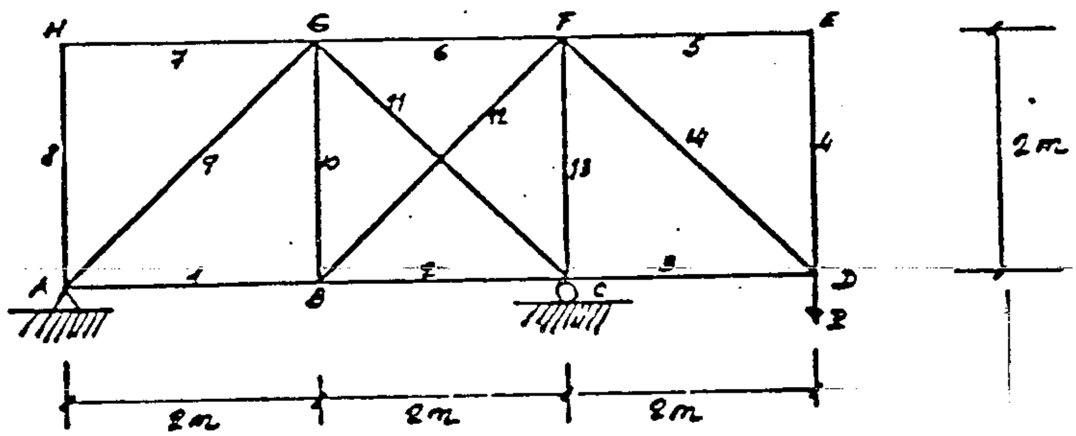
Asignatura : ANALISIS DE ESTRUCTURAS - METODOS NUMERICOS

Problema : 4

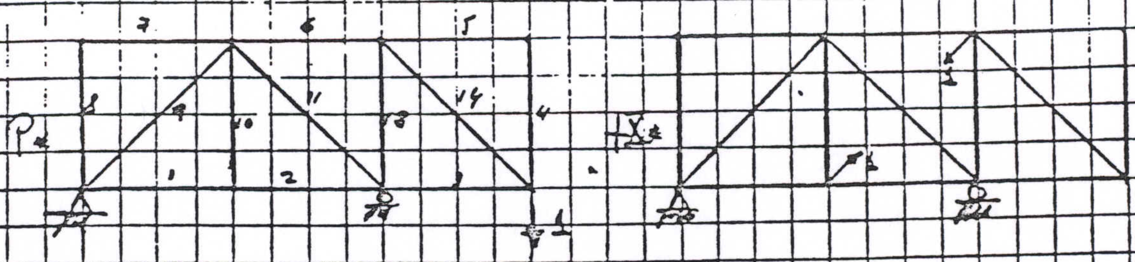
Sabiendo que la estructura de la figura se ha montado con la barra 12 un centímetro más corta; calcular la fuerza vertical P necesaria para que después del montaje el desplazamiento vertical del punto sea cero.

atos: $E = 2 \times 10^6$ Kg/cm^2

$A = 10$ cm^2 en todas las barras



PROBLEMA:



(II) (I)

$EA = 2 \times 10^{7}$

BARRAS	L_i	N_i^{I}	N_i^{II}	$L_i N_i^{II^2}/100$	$L_i N_i^{II} N_i^{I}/100$	$L_i N_i^{I^2}$
1	200	0	$-1/2$	$1/2$	0	0
2	200	$-\sqrt{2}/2$	$-1/2$	$1/2$	$\sqrt{2}/2$	1
3	200	0	-1	2	0	0
4	200	0	0	0	0	0
5	200	0	0	0	0	0
6	200	$-\sqrt{2}/2$	1	2	$-\sqrt{2}$	1
7	200	0	0	0	0	0
8	200	0	0	0	0	0
9	$200\sqrt{2}$	0	$\sqrt{2}/2$	$\sqrt{2}$	0	0
10	200	$-\sqrt{2}/2$	0	0	0	1
11	$200\sqrt{2}$	1	$-\sqrt{2}/2$	$\sqrt{2}$	-2	$2\sqrt{2}$
12	$200\sqrt{2}$	—	—	—	—	—
13	200	$-\sqrt{2}/2$	-1	2	$\sqrt{2}$	1
14	$200\sqrt{2}$	0	$\sqrt{2}$	$4\sqrt{2}$	0	0
				$\Sigma = 7+6\sqrt{2}$	$\Sigma = -2+\frac{\sqrt{2}}{2}$	$\Sigma = 4+2\sqrt{2}$

$V_0 = 0 \qquad 0 = P\,(7+6\sqrt{2}) + X\left(-2+\frac{\sqrt{2}}{2}\right)$

$$0 = P(7+6\sqrt{2}) + X\left(\frac{-4+\sqrt{2}}{2}\right) \Rightarrow \boxed{0 = 2(7+6\sqrt{2})P + (-4+\sqrt{2})X}$$

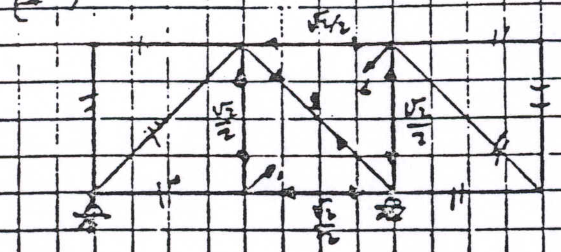
$$\delta_{12\,ab} = \left[P\left(-2+\frac{\sqrt{2}}{2}\right) + X\left(4+2\sqrt{2}\right)\right]\frac{1}{2\times10^{5}}$$

$$\delta_{12\,barra} = -\frac{X\,200\sqrt{2}}{2\times10^{7}} + \delta = -\frac{2X\sqrt{2}}{2\times10^{5}} + \delta = 1$$

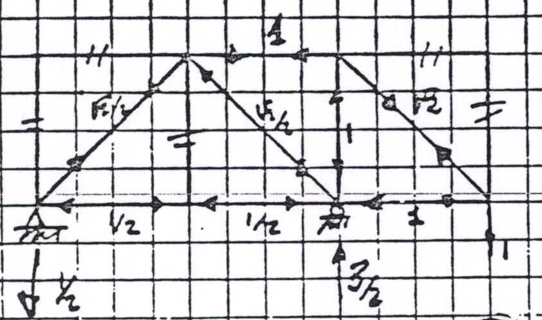
$$\left[P\,\frac{-4+\sqrt{2}}{2} + X\left(4+2\sqrt{2}\right) + 2\sqrt{2}X\right]\frac{1}{2\times10^{5}} = 1$$

$$\boxed{P\left(-4+\sqrt{2}\right) + 8X\left(1+\sqrt{2}\right) = 4\times10^{5}}$$

(I)



(II)



$$\left.\begin{array}{l} 2(7+6\sqrt{2})P + (-4+\sqrt{2})X = 0 \\ (-4+\sqrt{2})P + 8(1+\sqrt{2})X = 4\times10^5 \end{array}\right\} \Rightarrow X = -2\,\frac{(7+6\sqrt{2})}{-4+\sqrt{2}}\,P$$

sust. a la otra

$$P = \frac{4\times10^5}{\left[-4+\sqrt{2}+16\,\dfrac{(1+\sqrt{2})(7+6\sqrt{2})}{4-\sqrt{2}}\right]} = 1748,72\ \text{Kg} \Rightarrow \underline{\underline{1,7487\ T}}$$

$$X = +20944.8\ \text{Kg} = \underline{\underline{+20,945\ T}}$$

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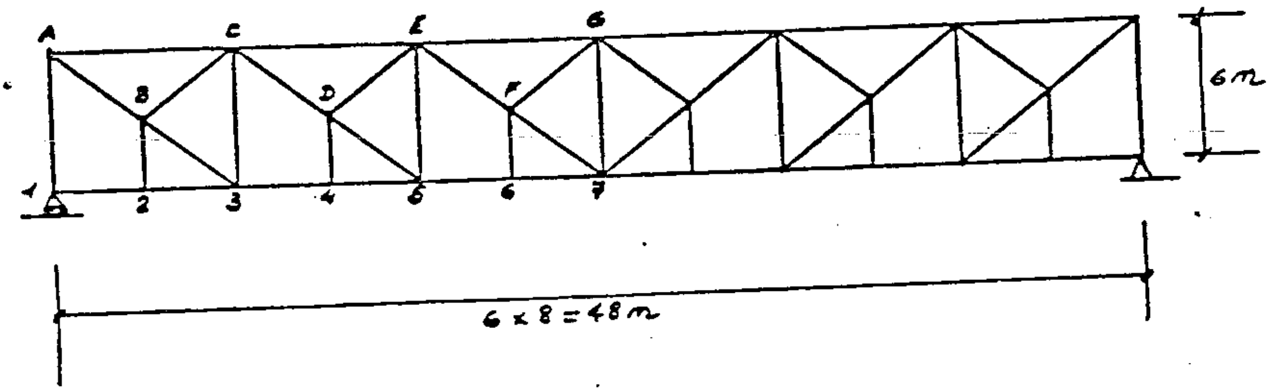
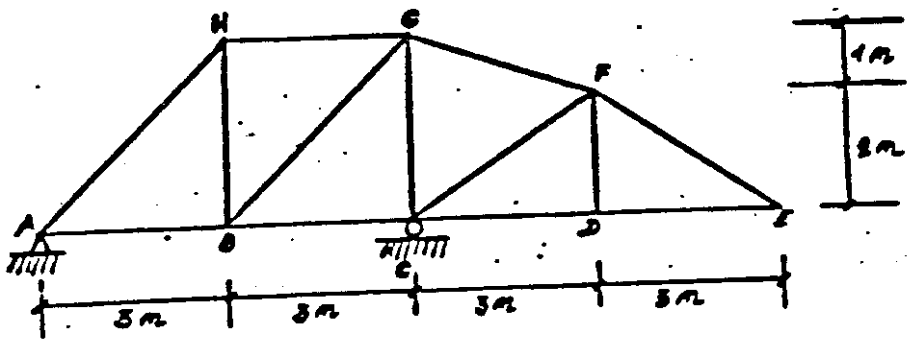
Asignatura: ANALISIS DE ESTRUCTURAS-METODOS NUMERICOS

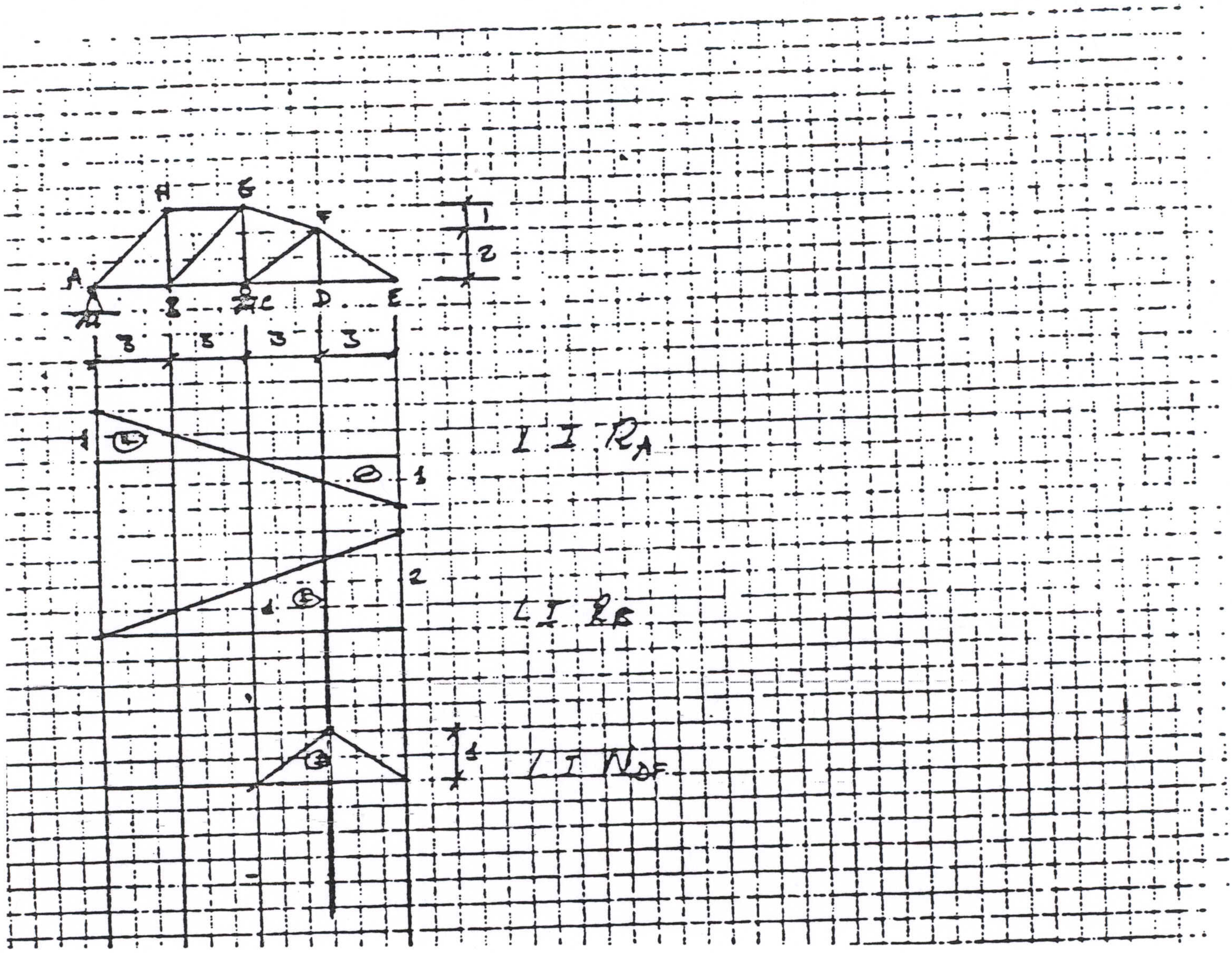
Problema 5 :

a) Calcular las líneas de influencia de los esfuerzos en las barras CE, 4D, DE, y E5 ,de la estructura representada en la figura, cuando una carga unidad recorre el cordón inferior(transmisión de modo indirecto).



Calcùlar las LINEAS DE INFLUENCIA de las reaccciones en A y C , y
esfuerzo de la barra DF ,cuando una carga unidad recorre el cordón
erior.

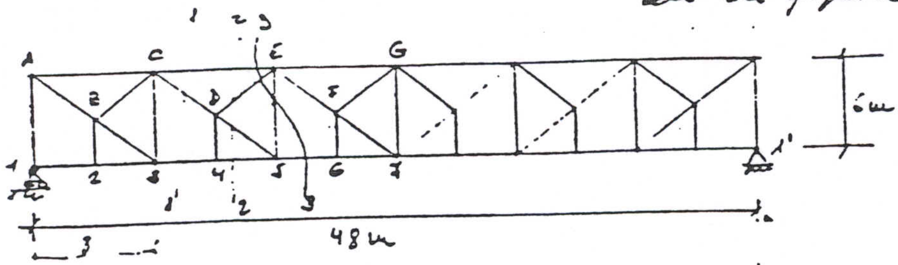


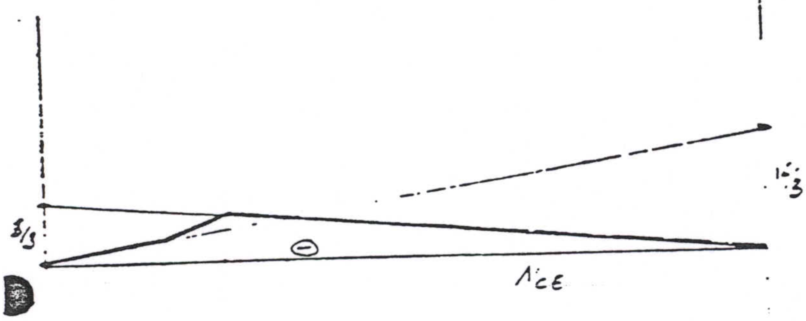


A
H
G
F
B
C
D
E
1
2
3
3
3
3
I I R_A
1
L I R_B
2
1
1
L I N_{DF}

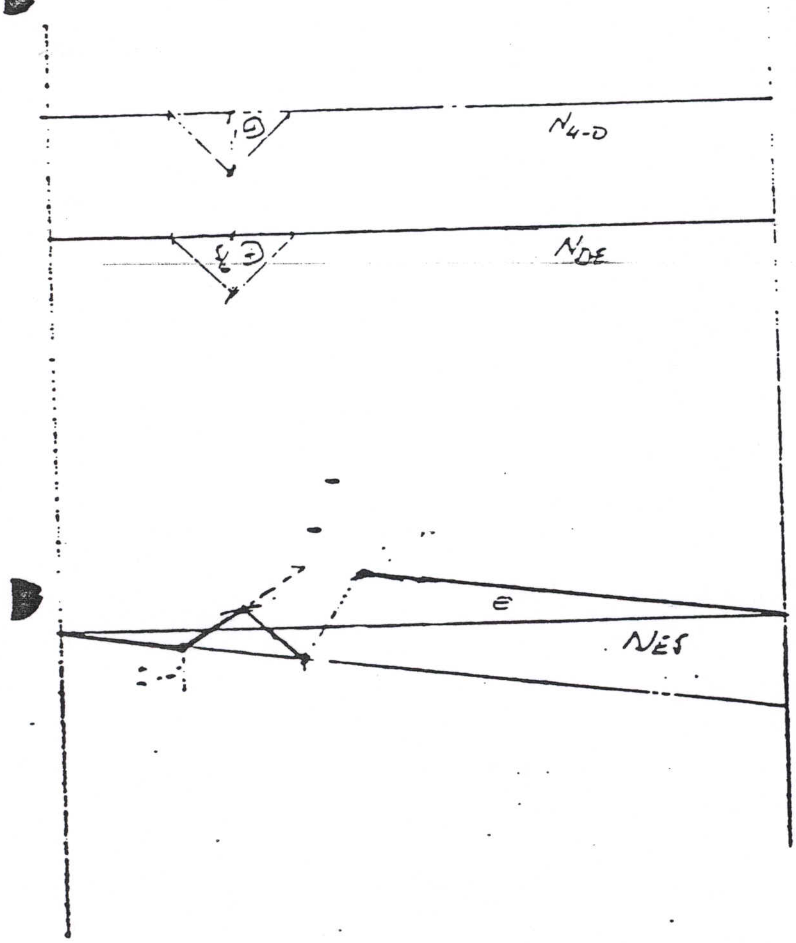
RES

PROBL 1: Calcular las líneas de influencia de los esfuerzos en las barras $\overline{CE}$, $\overline{4D}$, $\overline{DE}$ y $\overline{E5}$ de la estructura representada en la figura, cuando una carga unidad recorre el cordón inferior (Transmisión de modo indirecto).





Corte 1-1 $\Sigma M_5 = 0$
$$\begin{cases} N_{CE} \times 6 + V_1' \times 16 = 0 \;; & N_{CE} = -\frac{8}{3} V_1' \\ N_{CE} \times 6 + V_1 \times 32 = 0 \;; & N_{CE} = -\frac{16}{3} V_1 \end{cases}$$



Inmediato

Con el eq. de nudo 2

~~o también Corte 2-2, ΣM_5, para calcular~~

sabiendo que con la carga en 4

$N_{CE} = -2$ y $V_1' = \frac{1}{4}$

$N_{DE}\, 6 \cos\alpha + \frac{1}{4}\, 32 - 2 \times 6 = 0 \Rightarrow N_{DE} = \frac{5}{6}$

Corte 3-3:

Parte izq. $\Sigma F_y^{izq} = V_1 + N_{5E} + N_{DE}$ ~~sen α~~ $= 0$

$N_{5E} = -V_1 - N_{DE} \cos\alpha$

Parte dcha $\Sigma F_y^{dcha} = V_1' - N_{5E} - N_{DE} \operatorname{sen}\alpha = 0$

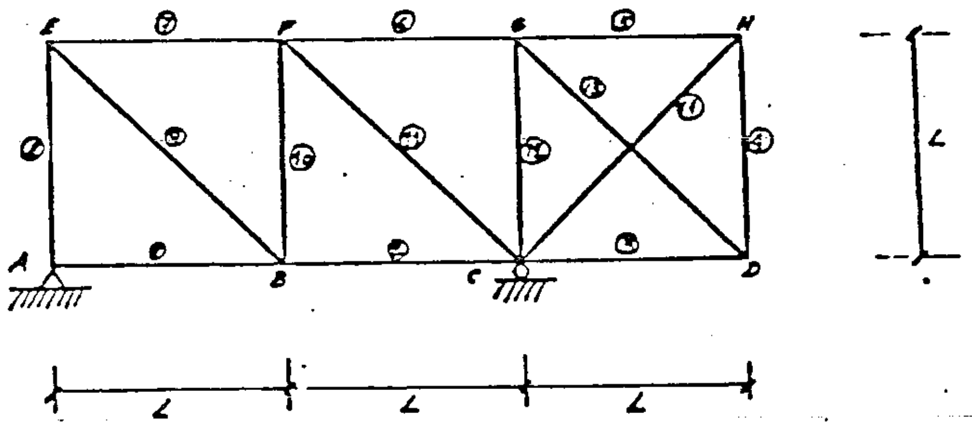
$N_{5E} = V_1' - N_{DE} \operatorname{sen}\alpha$

UNIVERSIDAD NACIONAL DE EDUCACION A DISTANCIA

Asignatura : ANALISIS DE ESTRUCTURAS - METODOS NUMERICOS

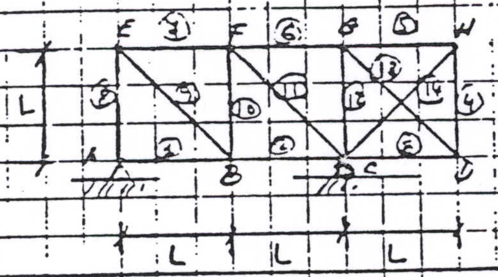
Problema : 6

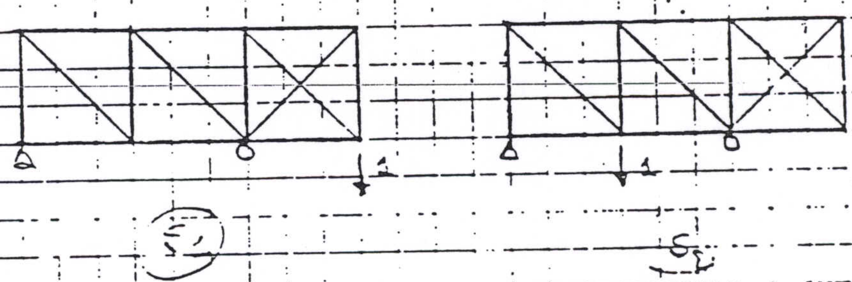
-Calcular en la estructura representada en la figura,las líneas de influencia de la reacción en A y de los esfuerzos axiles en las barras $\overline{EB}$ y $\overline{GD}$,cuando una carga unidad recorre el cordón inferior.



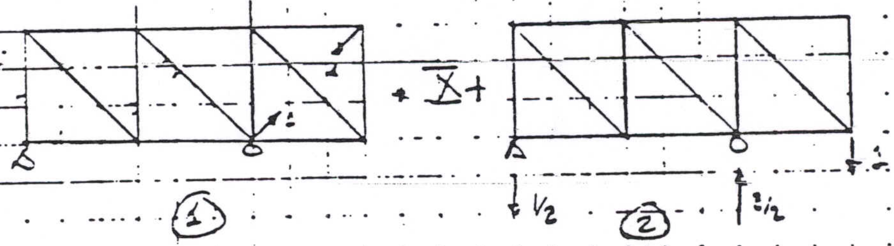
Reserva Sep.

PROBLEMA: Calcular en la estructura representada en la figura, las líneas de influencia de la reacción en A y de los esfuerzos axiles en las barras EB y GD, cuando una carga unidad recorre el cordón inferior.

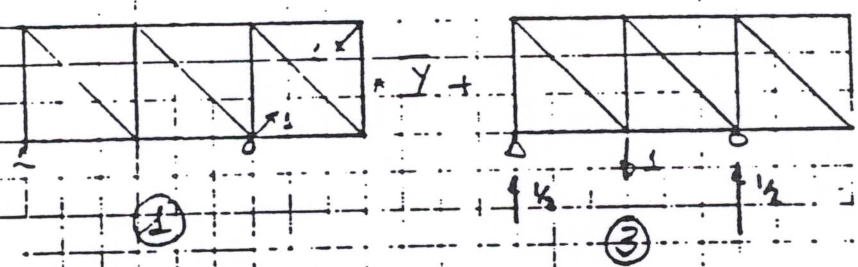




S_1 = ① $\cdot X +$ ②

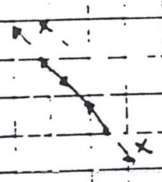


S_2 = ① $\cdot Y +$ ③



BARRA	l_i/L	$F_i^{(1)}$	$F_i^{(2)}$	$F_i^{(3)}$	$F_i^{(1)2}\, l_i/L$	$F_i^{(1)} F_i^{(2)}\, l_i/L$	$F_i^{(1)} F_i^{(3)}\, l_i/L$
1	1	0	0	0	0	0	0
2	1	0	$-1/2$	$1/2$	0	0	0
3	1	$-\sqrt{2}/2$	-1	0	$1/2$	$\sqrt{2}/2$	0
4	1	$-\sqrt{2}/2$	0	0	$1/2$	0	0
5	1	$-\sqrt{2}/2$	0	0	$1/2$	0	0
6	1	0	1	0	0	0	0
7	1	0	$1/2$	$-1/2$	0	0	0
8	1	0	$1/2$	$-1/2$	0	0	0
9	$\sqrt{2}$	0	$-\sqrt{2}/2$	$\sqrt{2}/2$	0	0	0
10	1	0	$1/2$	$1/2$	0	0	0
11	$\sqrt{2}$	0	$-\sqrt{2}/2$	$-\sqrt{2}/2$	0	0	0
12	1	$-\sqrt{2}/2$	-1	0	$1/2$	$\sqrt{2}/2$	0
13	$\sqrt{2}$	1	$\sqrt{2}$	0	$\sqrt{2}$	2	0
14	$\sqrt{2}$	—	—	—			
					$\Sigma = 2+\sqrt{2}$	$\Sigma = 2+\sqrt{2}$	0

(S1)



$$u_{14}^{I} = -X \frac{L\sqrt{2}}{AE} = -X\sqrt{2}\frac{L}{AE}$$

$$v_{14}^{II} = \sum \left(X N_i^{(1)} + N_i^{(2)} \right) N_i^{(1)} \frac{l_i}{AE}$$

$$u_{14}^{I} = v_{14}^{II}$$

$$-X\sqrt{2}\frac{L}{AE} = \frac{L\,\Sigma}{AE}\; X N_i^{(1)2} \frac{l_i}{L} + N_i^{(1)} N_2^{(2)} \frac{l_i}{L}$$

$$-X\sqrt{2} = X\left(2+\sqrt{2}\right) + 2+\sqrt{2} \Rightarrow \boxed{X = -\frac{2+\sqrt{2}}{2+2\sqrt{2}}}$$

(S2) Analogamente: $-X\sqrt{2} = X\left(2+\sqrt{2}\right) + 0 \Rightarrow \boxed{Y = 0}$

(M)

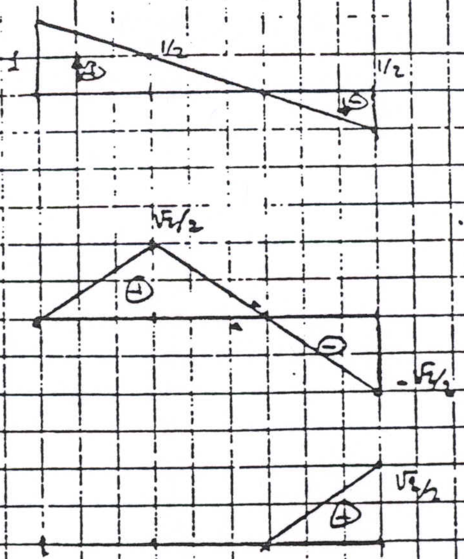
	S_1	S_2
R_A	$1/2$	$1/2$
$N_{EB} = N_9$	$X N_9^{(1)} + N_9^{(2)} = \sqrt{2}/2$	$\sqrt{2}/2$
$N_{GD} = N_{13}$	$X N_{13}^{(1)} + N_{13}^{(2)} = -\frac{2+\sqrt{2}}{2+2\sqrt{2}} + \sqrt{2} = \frac{\sqrt{2}}{2}$	0

③

LI R_A:

LI $N_{EB} = N_9$:

LI $N_{GD} = N_{13}$:



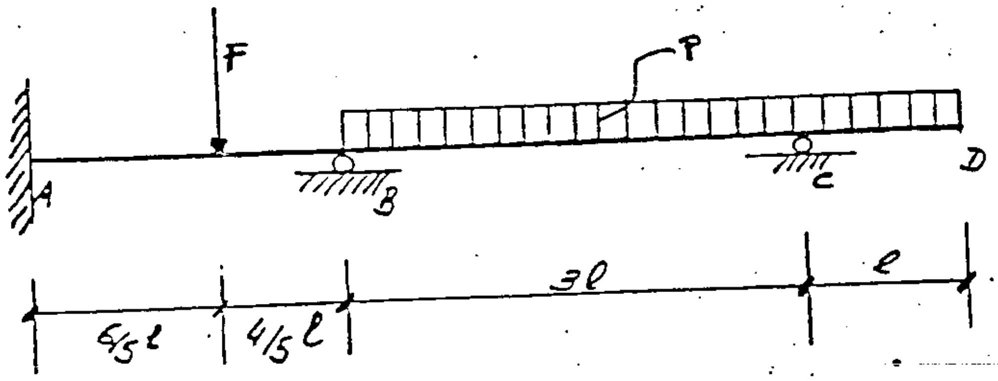
UNIVERSIDAD NACIONAL DE EDUCACION A DISTANCIA

Asignatura: ANALISIS DE ESTRUCTURAS-METODOS NUMERICOS

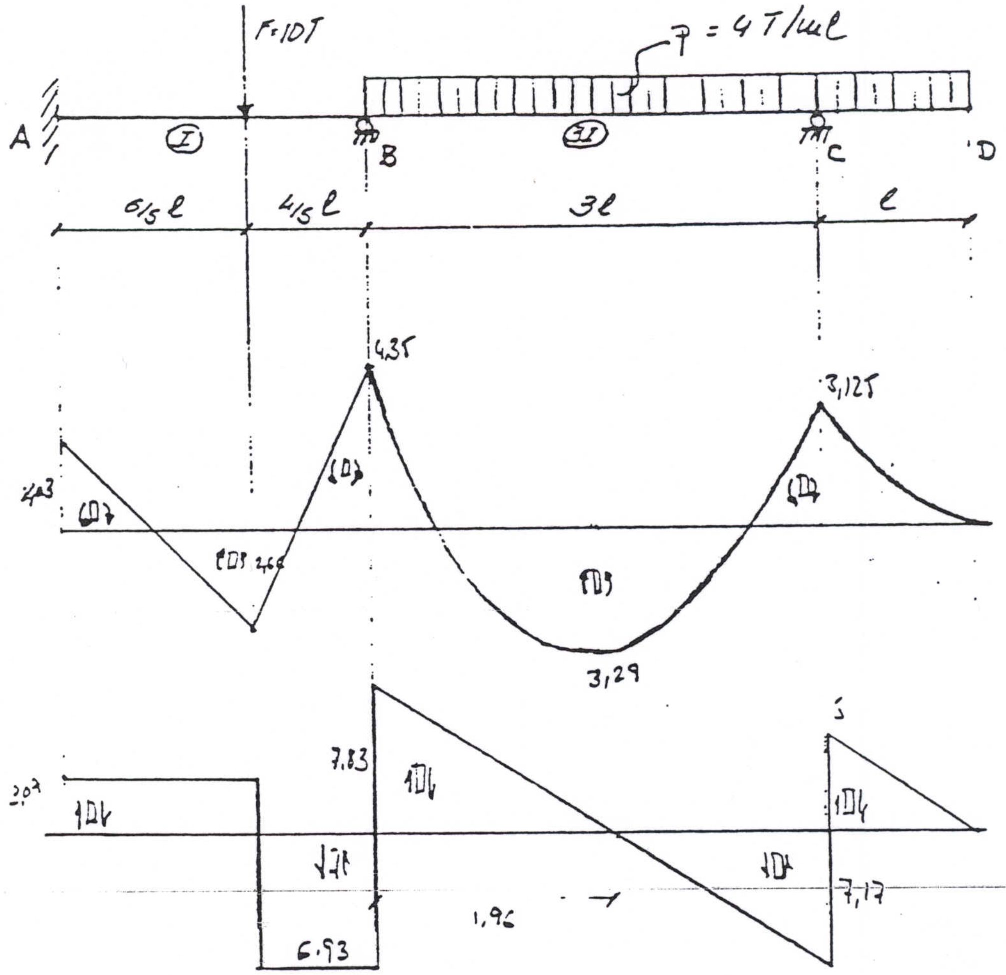
Problema : 1

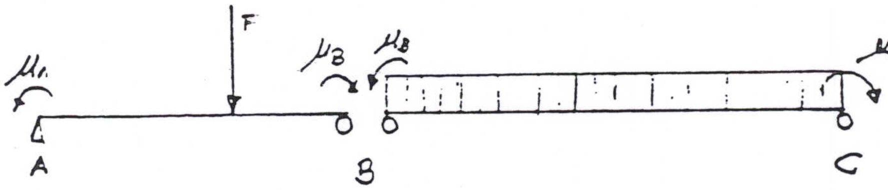
Determinar las leyes de momentos flectores, cortantes, y reacciones de la viga de la figura.

Sabiendo que : l=1,25 m , que la inercia del tramo AB es I , y la inercia del tramo BC es 3I . La fuerza puntual F es de 10 T, y la carga distribuida es de P=4 T/m



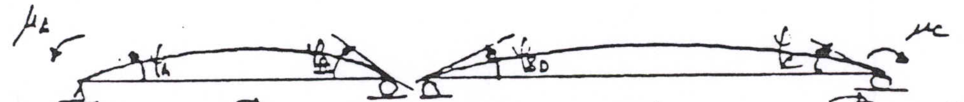
$l = 1{,}25\ m$





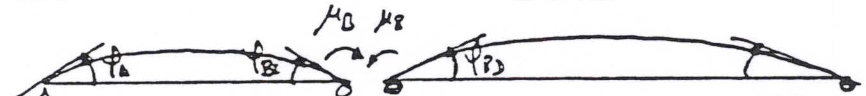
$$\mu_C = \frac{pl^2}{2} = \frac{4 \times 1{,}25^2}{2} = 3{,}125$$

1°



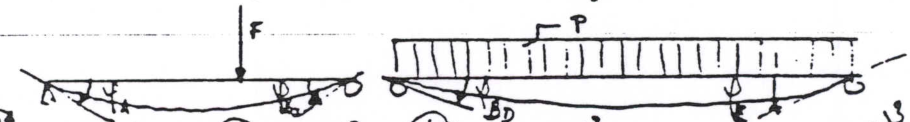
$$\varphi_A = \frac{\mu_A (2l)}{3EI} \qquad \varphi_{B_I} = \frac{\mu_A (2l)}{6EI} \qquad \varphi_{BD} = \frac{\mu_C (3l)}{6E(3I)} \qquad \varphi_C = \frac{\mu_C (3l)}{3E(3I)}$$

2°



$$\varphi_A = \frac{\mu_B (2l)}{6EI} \qquad \varphi_{B_I} = \frac{\mu_B (2l)}{3EI} \qquad \varphi_{BD} = \frac{\mu_B (3l)}{3E(3I)} \qquad \varphi_C = \frac{\mu_B (3l)}{6E(3I)}$$

3°



$$\varphi_A = \frac{28Fl^2}{125EI} \qquad \varphi_{B_I} = \frac{32Fl^2}{125EI} \qquad \varphi_{BD} = \frac{p(3l)^3}{24E(3I)} \qquad \varphi_C = \frac{p(3l)^3}{24E(3I)}$$

NUDO A: Como es un empotramiento, el giro será nulo.

$$\overset{\frown}{\varphi}_{A_1} + \overset{\frown}{\varphi}_{A_2} + \overset{\frown}{\varphi}_{A_3} = 0$$

$$\frac{\mu_A \, 2\ell}{3EI} + \frac{\mu_B \, 2\ell}{6EI} - \frac{28 F \ell^2}{125 EI} = 0$$

$$\boxed{2\mu_A + \mu_B = \frac{84 F \ell}{125} = \frac{84 \times 10 \times 1{,}25}{125} = 8{,}4}$$

NUDO B: Por continuidad, el giro a la derecha y a la izquierda iguales.

$$\overset{\frown}{\varphi}_{BI} = -\overset{\frown}{\varphi}_{BD}$$

$$\overset{\frown}{\varphi}_{BI} = \overset{\frown}{\varphi}_{BI_1} + \overset{\frown}{\varphi}_{BI_2} + \overset{\frown}{\varphi}_{BI_3} =$$

$$= -\frac{\mu_A \, 2\ell}{6EI} - \frac{\mu_B \, 2\ell}{3EI} + \frac{32 F\ell^2}{125 EI} = \frac{\ell}{EI}\left[\frac{32 F\ell}{125} - \frac{2\mu_B + \mu_A}{3}\right]$$

$$\overset{\frown}{\varphi}_{BD} = \overset{\frown}{\varphi}_{BD_1} + \overset{\frown}{\varphi}_{BD_2} + \overset{\frown}{\varphi}_{BD_3} =$$

$$= -\frac{\mu_C \, 3\ell}{6E\,3I} - \frac{\mu_B \, 3\ell}{3E\,3I} + \frac{P\,27\ell^3}{24 E\,3I} = \frac{\ell}{EI}\left[\frac{3P\ell^2}{8} - \frac{2\mu_B + \mu_C}{6}\right]$$

$$\frac{32 F\ell}{125} - \frac{2\mu_B + \mu_A}{3} = -\frac{3P\ell^2}{8} + \frac{2\mu_B + \mu_C}{6}$$

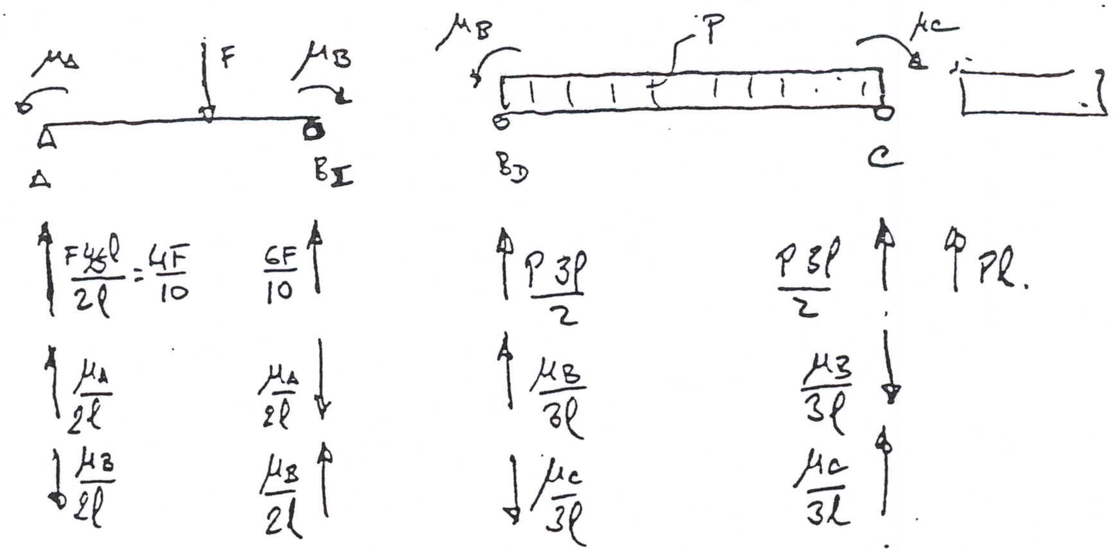
$$\boxed{3\mu_B + \mu_A = \frac{96 F\ell}{125} + \frac{7P\ell^2}{8}} = \frac{96 \times 10 \times 1{,}25}{125} + \frac{7 \times 4 \times \overline{1{,}25}^2}{8} = 15{,}07$$

$$\left.\begin{array}{l} 2\mu_A + \mu_B = 8{,}4 \\ \mu_A + 3\mu_B = 15{,}07 \end{array}\right\} \Rightarrow$$

$\mu_B = 4{,}35$ mT
$\mu_A = 2{,}03$ mT

178-100

PROBLEMA 2



$$\uparrow R_A = \frac{4 \times F}{10} + \frac{M_A}{2l} - \frac{M_B}{2l} = 4 + \frac{2{,}03}{2 \times 1{,}25} - \frac{4{,}35}{2 \times 1{,}25} = \underline{\underline{3{,}07\ T}}$$

$$\uparrow R_B = \left[6 + \frac{4{,}35}{2 \times 1{,}25} - \frac{2{,}03}{2 \times 1{,}25} \right] + \left[\frac{4 \times 3 \times 1{,}25}{2} + \frac{4{,}35}{3 \times 1{,}25} - \frac{3{,}125}{3 \times 1{,}25} \right] =$$

$$= 6{,}93 + 7{,}83 = \underline{\underline{14{,}76\ T}}$$

$$\uparrow R_C = \left[\frac{4 \times 3 \times 1{,}25}{2} + \frac{3{,}125}{3 \times 1{,}25} - \frac{4{,}35}{3 \times 1{,}25} \right] + 4 \times 1{,}25 =$$

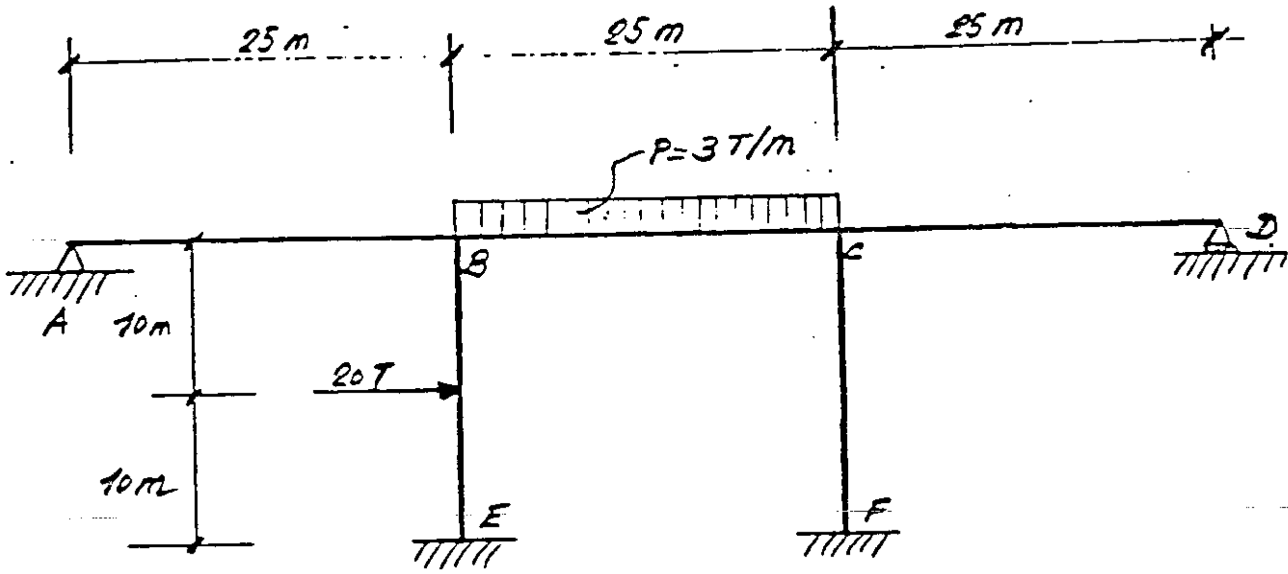
$$= 7{,}17 + 5 = \underline{\underline{12{,}17\ T}}$$

UNIVERSIDAD NACIONAL DE EDUCACION A DISTANCIA

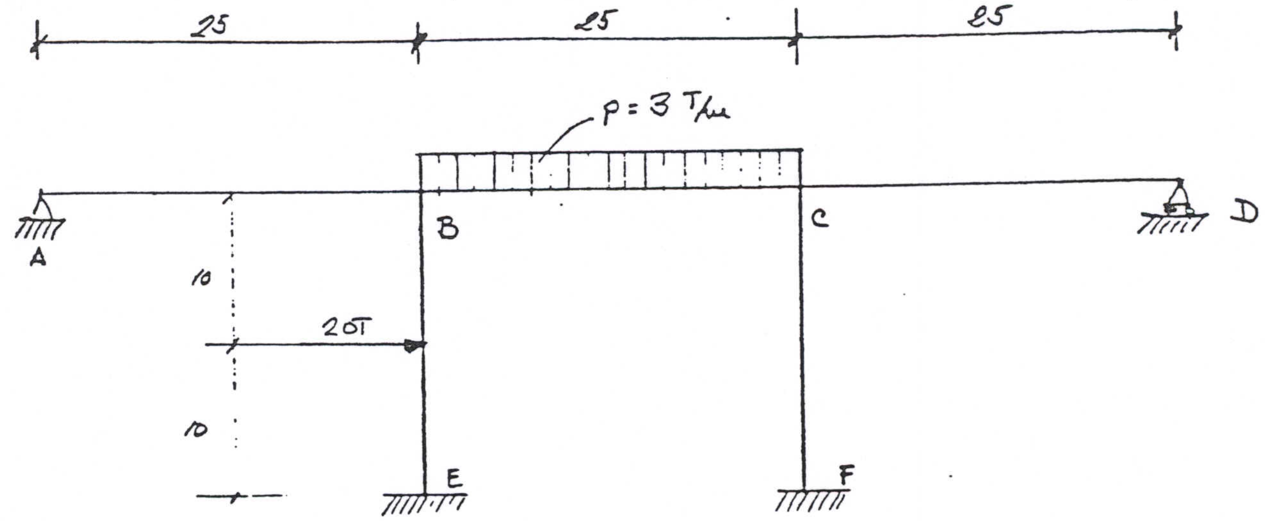
Asignatura: ANALISIS DE ESTRUCTURAS -METODOS NUMERICOS

Problema 2:

Calcular las Leyes de momentos flectores en el pórtico intraslacional de barras iguales de la figura, así como las reacciones y estudiar la deformada



Problema 47



Momentos de empotramiento:

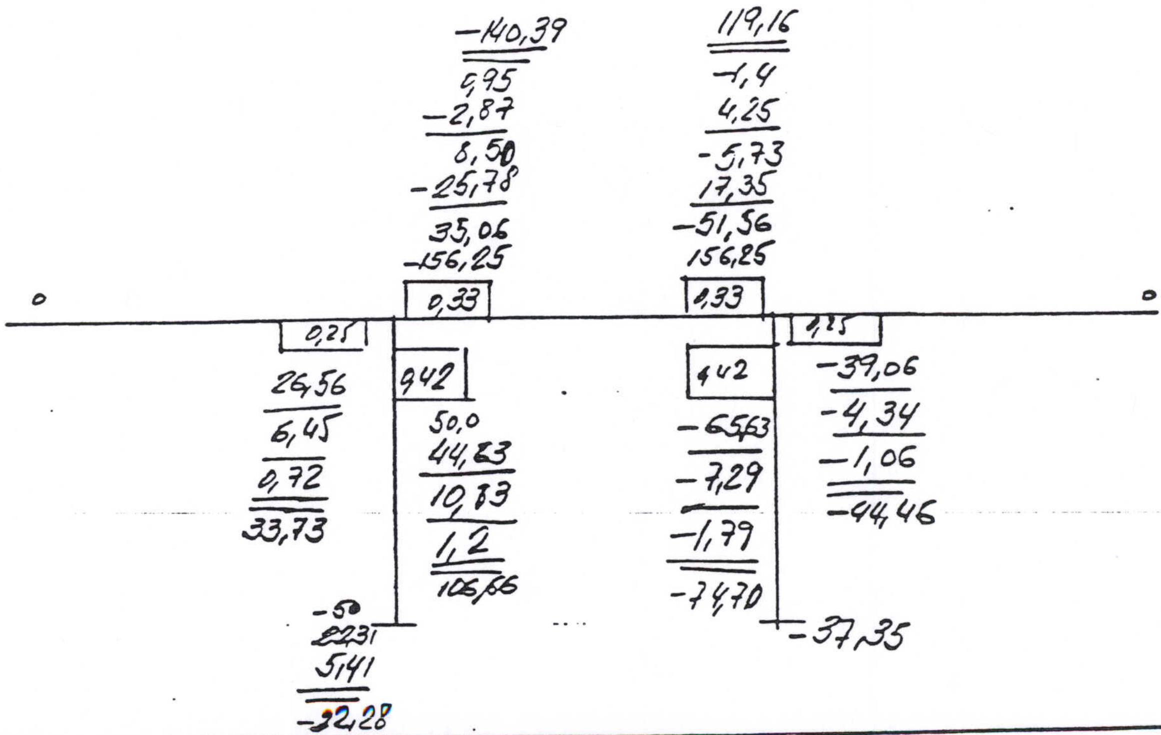
$$M_{BC} = -\frac{pL^2}{12} = -\frac{3 \times 25^2}{12} = -156,25 \text{ mT} \quad ; \quad M_{CB} = 156,25 \text{ mT}$$

$$M_{EB} = -\frac{PL}{8} = -\frac{20 \times 20}{8} = -50,0 \text{ mT} \quad ; \quad M_{BE} = 50,0 \text{ mT}$$

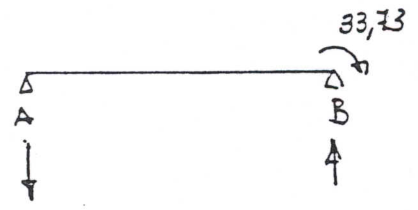
Rigideces:

$$K_{AB} = \frac{3EI}{25} \; , \; K_{BC} = \frac{4EI}{25} \; , \; K_{CD} = \frac{3EI}{25}$$

$$K_{BE} = K_{CF} = \frac{4EI}{20}$$

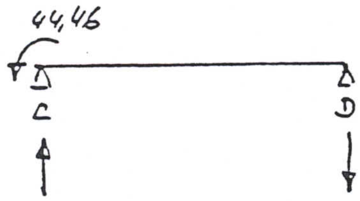


REACCIONES:

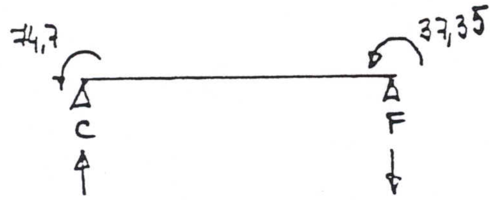


⇒ $R_A^V \downarrow = 1{,}35\ T$

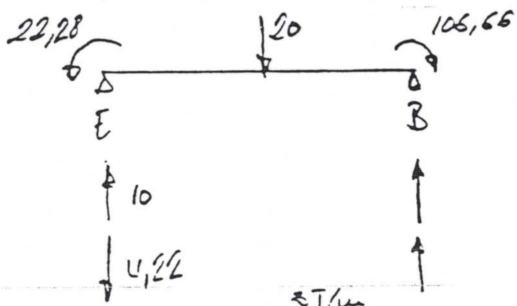
Reacción



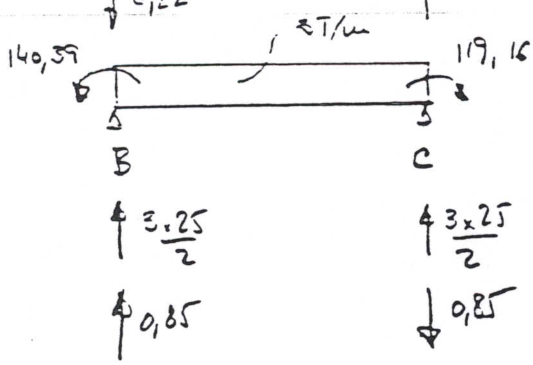
⇒ $R_D^V \downarrow = 1{,}78\ T$



⇒ $R_F^V \downarrow = 5{,}6\ T$

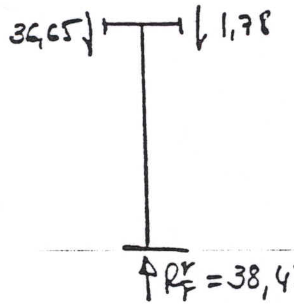


⇒ $R_E^V \uparrow = 5{,}78\ T$



⇒ $R_C \uparrow = 36{,}65\ T$

Reacción vertical en F:

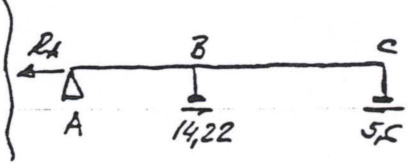


$R_F^V \uparrow = 38{,}43$

Reacción vertical en E:

$R_E^V \uparrow = 3 \times 25 + 1{,}35 + 1{,}78 - 38{,}43 = 39{,}70$

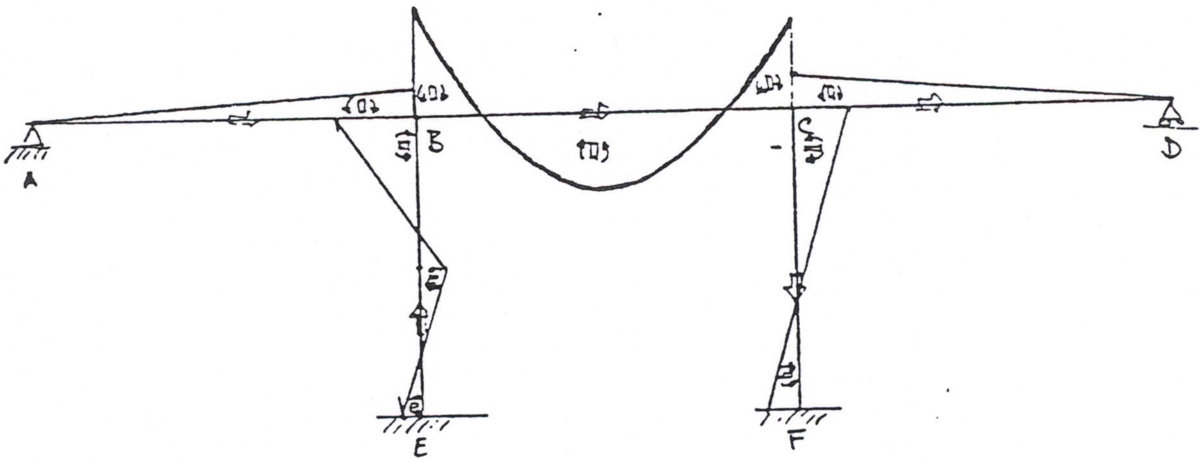
Reacción horizontal en A:

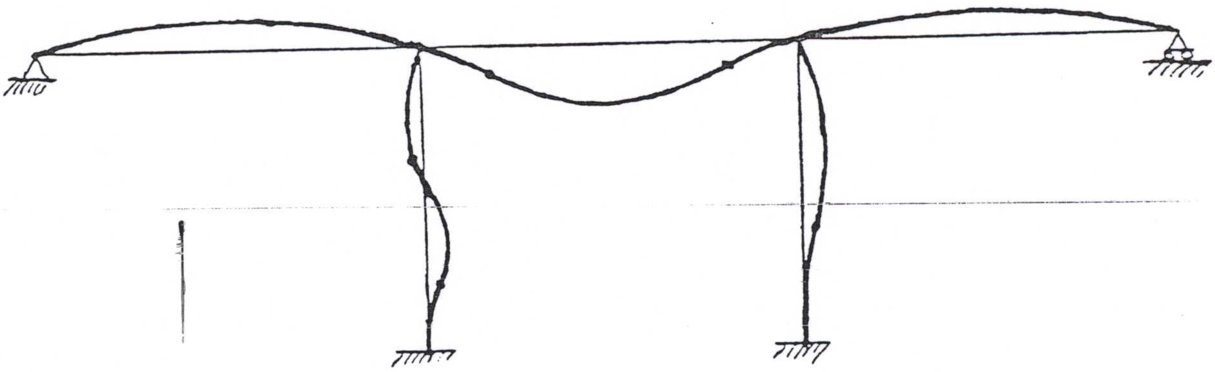


$R_A = 14{,}22 - 5{,}6 = 8{,}6$

NOTA: Como comprob. Tomamos momentos respecto de E:

$$-22{,}28 - 1{,}35 \times 25 + 1{,}78 \times 50 - 38{,}43 \times 25 - 37{,}35 + 3 \times \frac{25^2}{2} + 20 \times 10 - 8{,}62 \times 20 \simeq 0$$


B
C
A
D
E
F

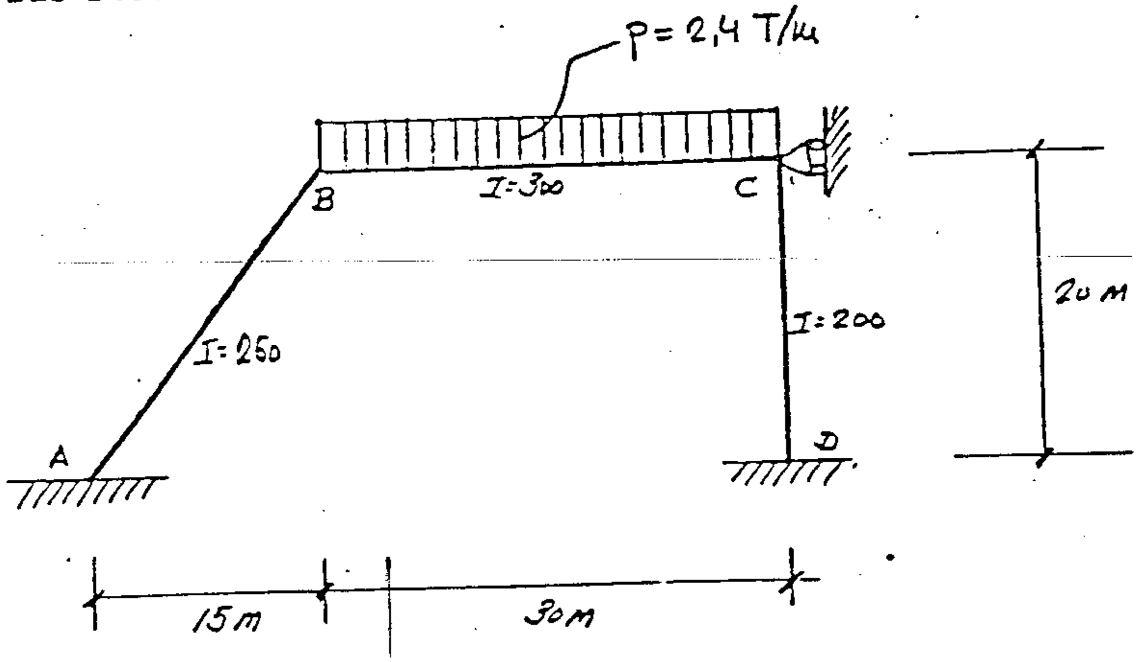


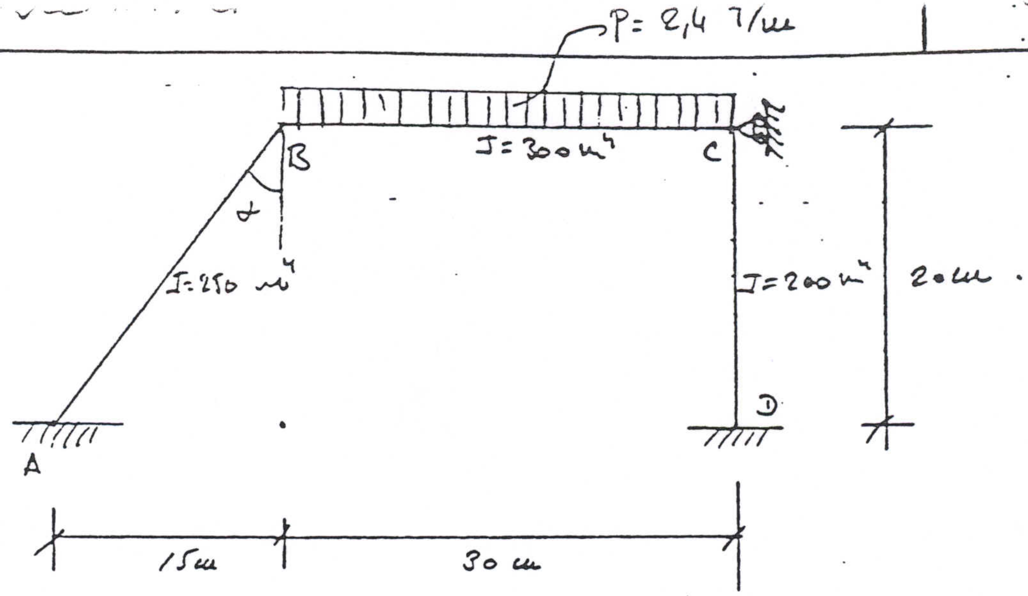
UNIVERSIDAD NACIONAL DE EDUCACION A DISTANCIA

Asignatura: ANALISIS DE ESTRUCTURAS-METODOS NUMERICOS

Probbema : 3

En la estructura intraslacional de la figura, determinar las leyes de momentos flectores, esfuerzos cortantes, axiles, así como las reacciones.



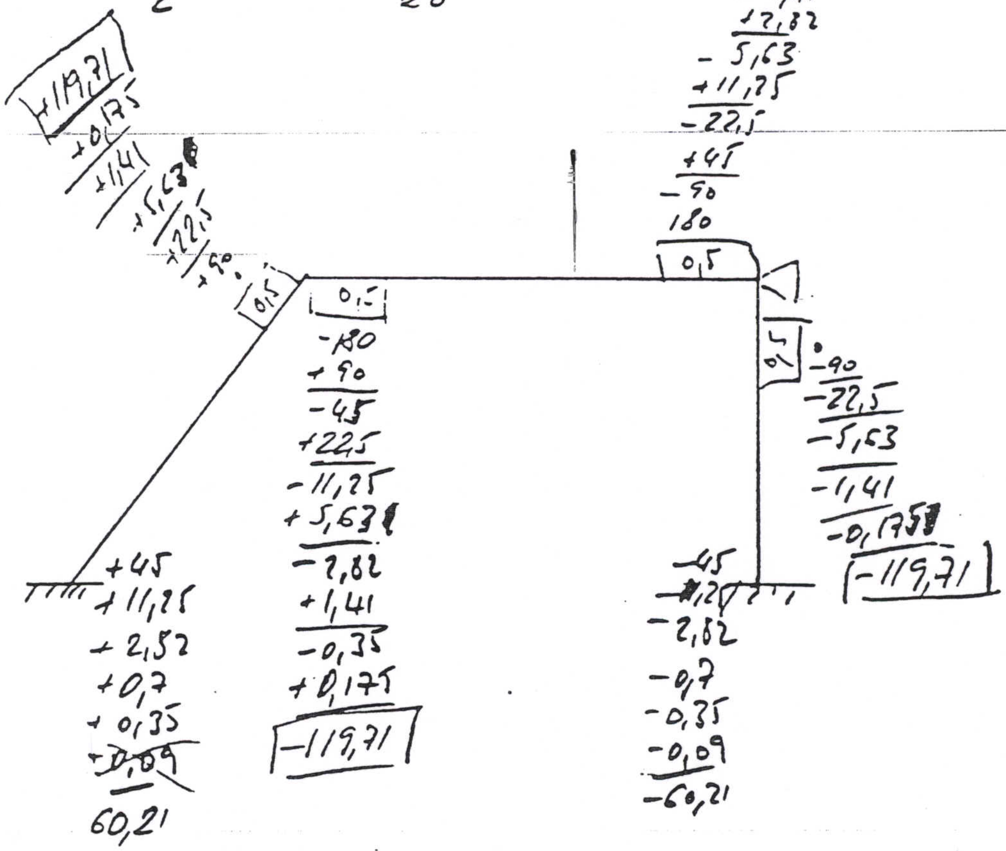


$$K_{AB} = \frac{4EI}{L} = \frac{4E\,250}{\sqrt{15^2+20^2}} = 40E$$

$$M_B = -\frac{pl^2}{12} = \frac{2{,}4 \times 30^2}{12} = -180$$

$$K_{BC} = \frac{4EI}{L} = \frac{4E\,300}{30} = 40E$$

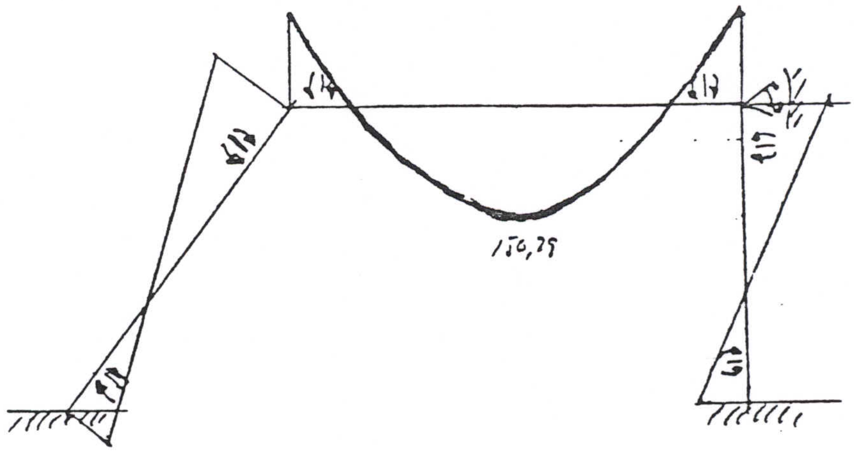
$$K_{CD} = \frac{4EI}{L} = \frac{4E\,200}{20} = 40E$$



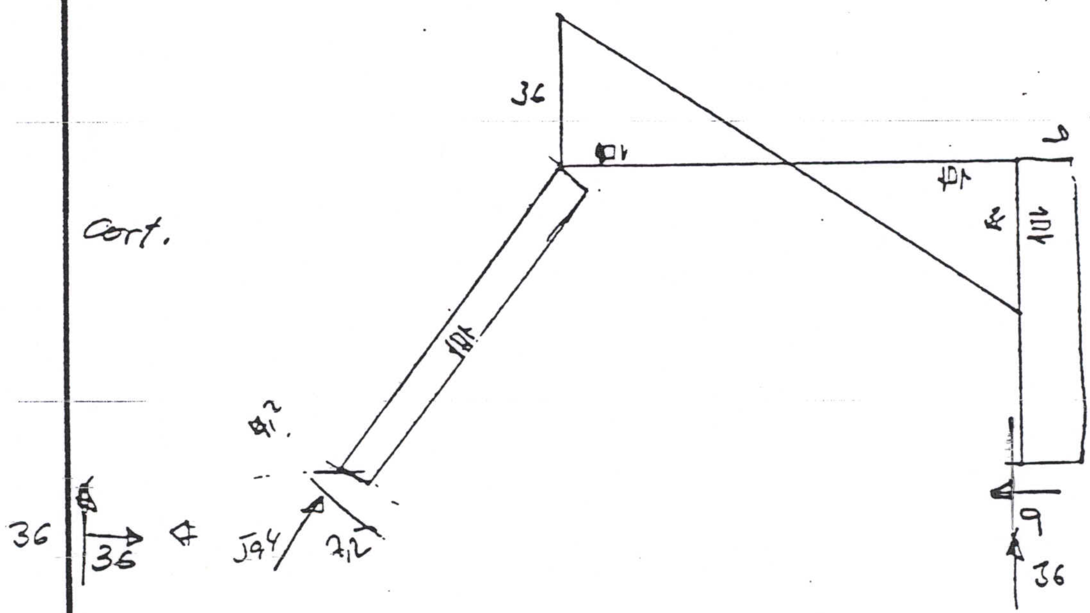
$$\alpha = 36{,}8699 \Rightarrow \begin{cases} \operatorname{sen}\alpha = 0{,}6 \\ \cos\alpha = 0{,}8 \end{cases}$$

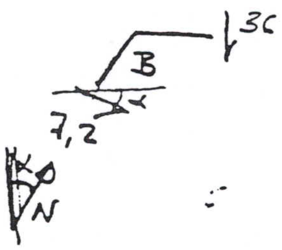
PROBLEMA. 6

Flectores.



Cort.

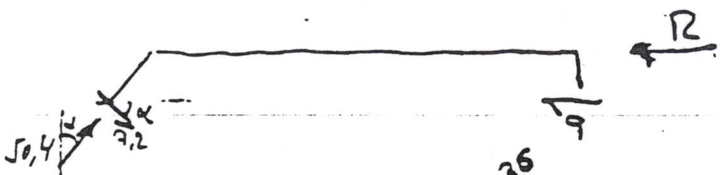




$$N\cos\alpha - 7{,}2\,\operatorname{sen}\alpha - 36 = 0$$

$$N \times 0{,}8 = 7{,}2 \; 0{,}6 + 36$$

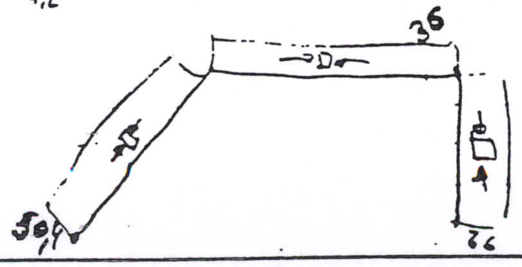
$$\boxed{N = 50{,}4}$$



$$R = 50{,}4 \times 0{,}6 + 7{,}2 \times 0{,}8 - 9 =$$

$$= \underline{27}$$

Axiles:



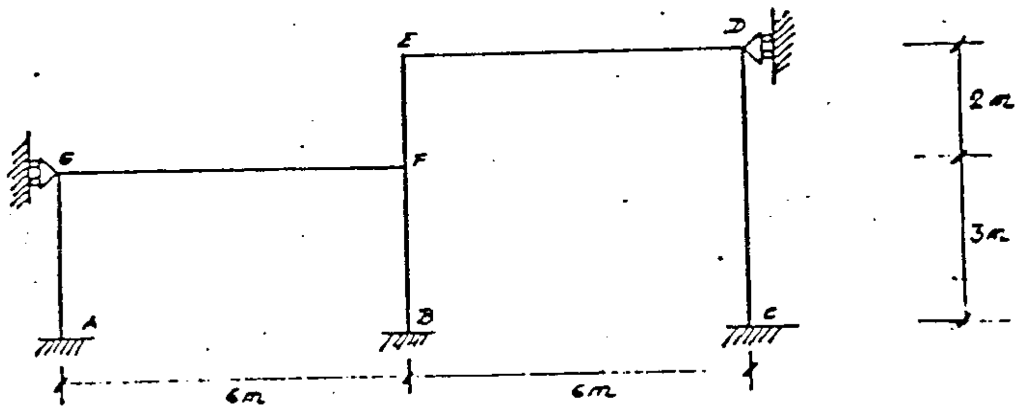
Problema : 4

- Hallar las reacciones horizontales en G y en D de la estructura representada en la figura, si el apoyo B sufre un descenso de 8 mm.

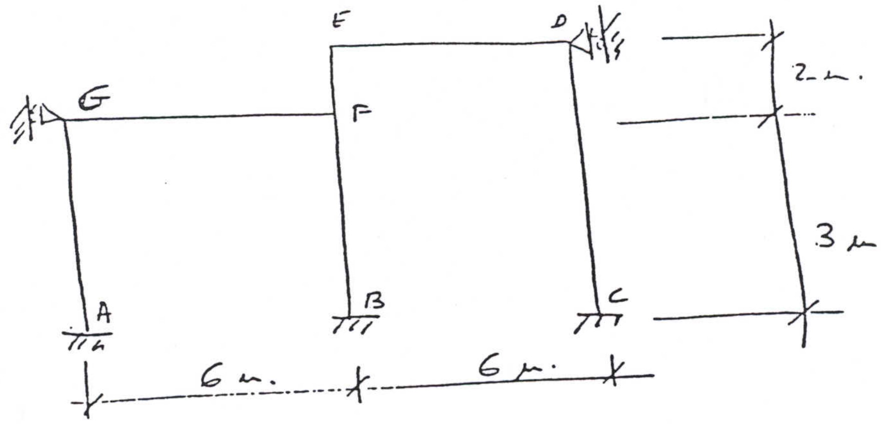
DATOS: E x I en las barras horizontales (DE y FG) = 9×10^3 Tm^2

E x I en las barras AG, BF y EF = $0,651 \times 10^3$ Tm^2

E x I en la barra CD = $2,667 \times 10^3$ Tm^2



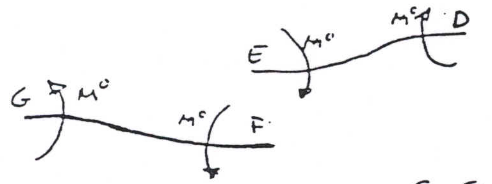
UD. 4 — Problema Nº 4 Hoja 1/2



Barra DE y FG — $EI = 9 \cdot 10^3$ t.m^2 = $9 \cdot 10^3$ t. 100^2 cm^2 = $9 \cdot 10^7$ t.cm^2

" AG, BF y EF — $EI = 0{,}651 \cdot 10^3$ t.m^2 = $0{,}651 \cdot 10^3$ t. 10^4 cm^2 = $0{,}651 \cdot 10^7$ t.cm^2

" CD — $EI = 2{,}667 \cdot 10^3$ t.m^2 = $2{,}667 \cdot 10^3$ t. 10^4 cm^2 = $2{,}667 \cdot 10^7$ t.cm^2



$$M^{\circ} = \frac{6EI}{L^2}\Delta = \frac{6 \cdot 9 \cdot 10^7 \text{ t.cm}^2}{(600)^2 \text{ cm}^2} \cdot 0{,}8 \text{ cm} = 1200 \text{ t.cm} = 12 \text{ t.m.}$$

$$K_{GF} = 4\frac{EI}{\ell} = \frac{4 \cdot 9 \cdot 10^7 \text{ t.cm}^2}{600 \text{ cm}} = 6 \cdot 10^5$$

$$K_{GA} = 4\frac{EI}{\ell} = \frac{4 \cdot 0{,}651 \cdot 10^7}{300} = 0{,}868 \cdot 10^5$$

$r_{GF} = 0{,}87$

$r_{GA} = 0{,}13$

$K_{FG} = 6 \cdot 10^5$

$K_{FB} = 0{,}868 \cdot 10^5$

$$K_{FE} = \frac{4 \cdot 0{,}651 \cdot 10^7}{200} = 1{,}302 \cdot 10^5$$

$r_{FG} = 0{,}73$

$r_{FB} = 0{,}11$

$r_{FE} = 0{,}16$

$K_{DE} = 6 \cdot 10^5$

$$K_{DC} = \frac{4 \cdot 2{,}667 \cdot 10^7}{500} = 2{,}1336 \cdot 10^5$$

$r_{DE} = 0{,}74$

$r_{DC} = 0{,}26$

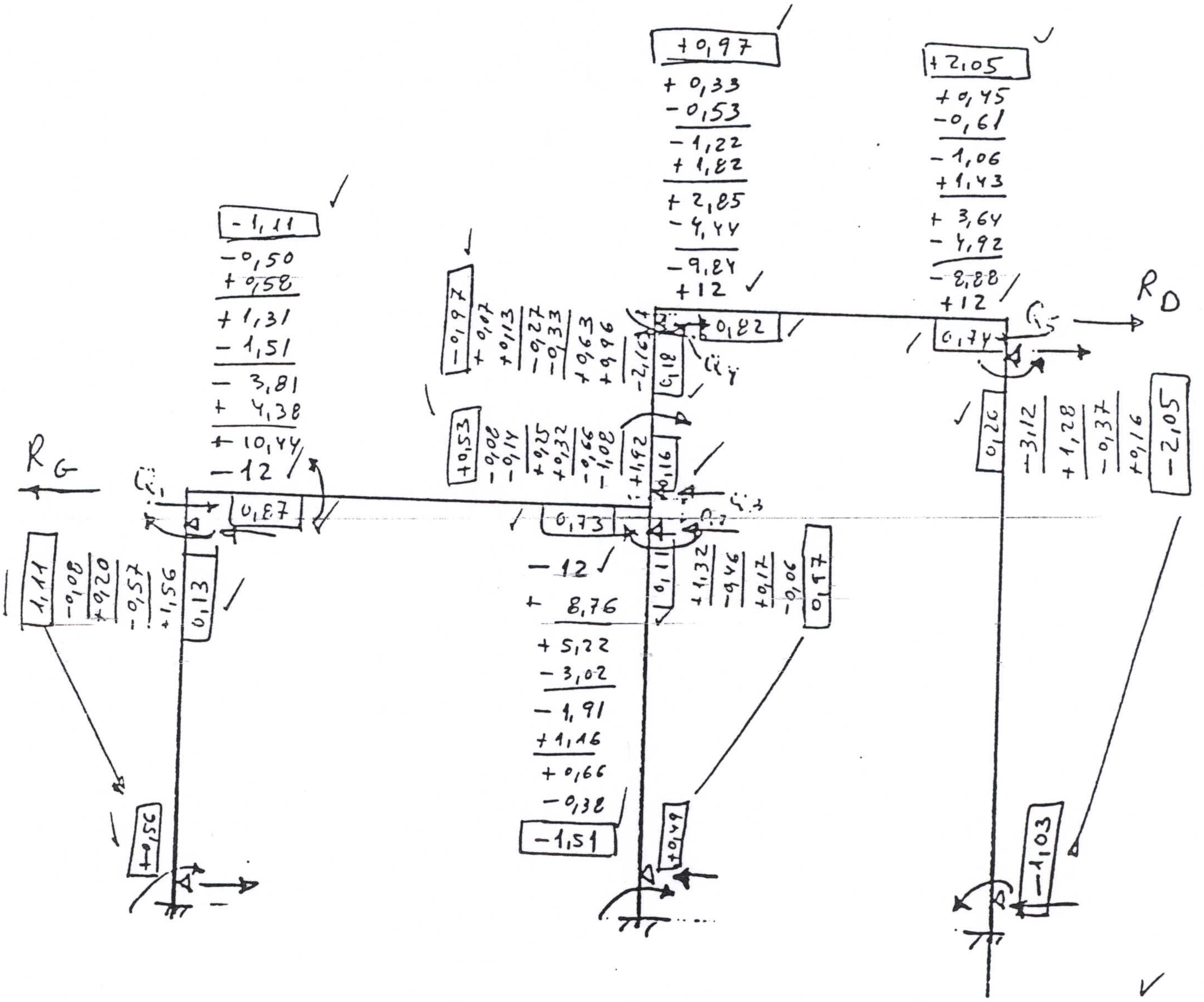
$K_{ED} = 6 \cdot 10^5$

$K_{EF} = 1{,}302 \cdot 10^5$

$r_{ED} = 0{,}82$

$r_{EF} = 0{,}18$

UD 4 — Problema 4 — Hoja 2/2



$$R_G = +Q_1 + Q_2 + Q_3 = + \frac{1{,}11 + 0{,}56}{3} + \frac{0{,}49 + 0{,}97}{3} + \frac{0{,}97 - 0{,}53}{2} = \boxed{1{,}26\ tn = R_G}$$

$$R_D = +Q_4 + Q_5 = + \frac{0{,}97 - 0{,}53}{2} + \frac{2{,}05 + 1{,}03}{5} = \boxed{~~1{,}76~~\ tn = R_D}$$

0,926

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Asignatura: ANALISIS DE ESTRUCTURAS-METODOS NUMERICOS

Problema: 5

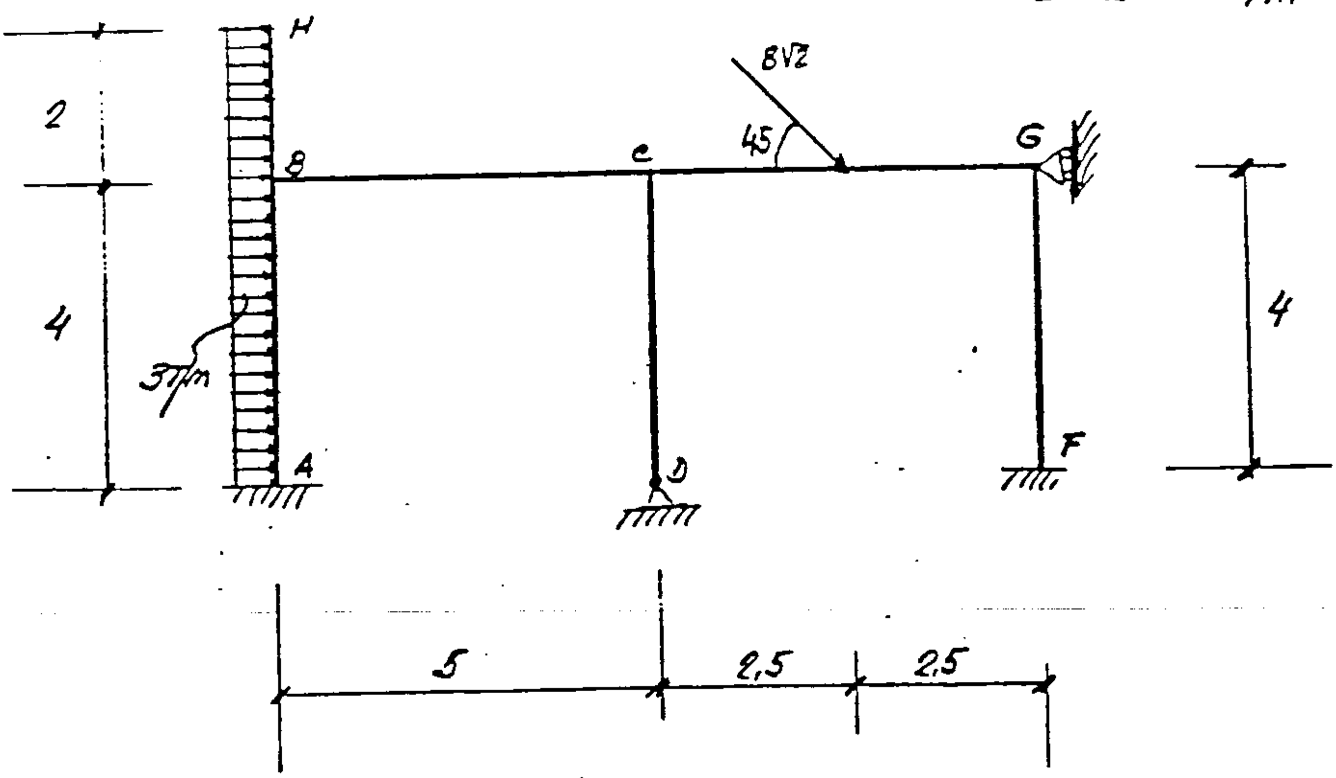
Calcular las reacciones, leyes de momentos flectores, esfuerzos cortantes y axiles, en el pórtico de la figura. Sabiendo:

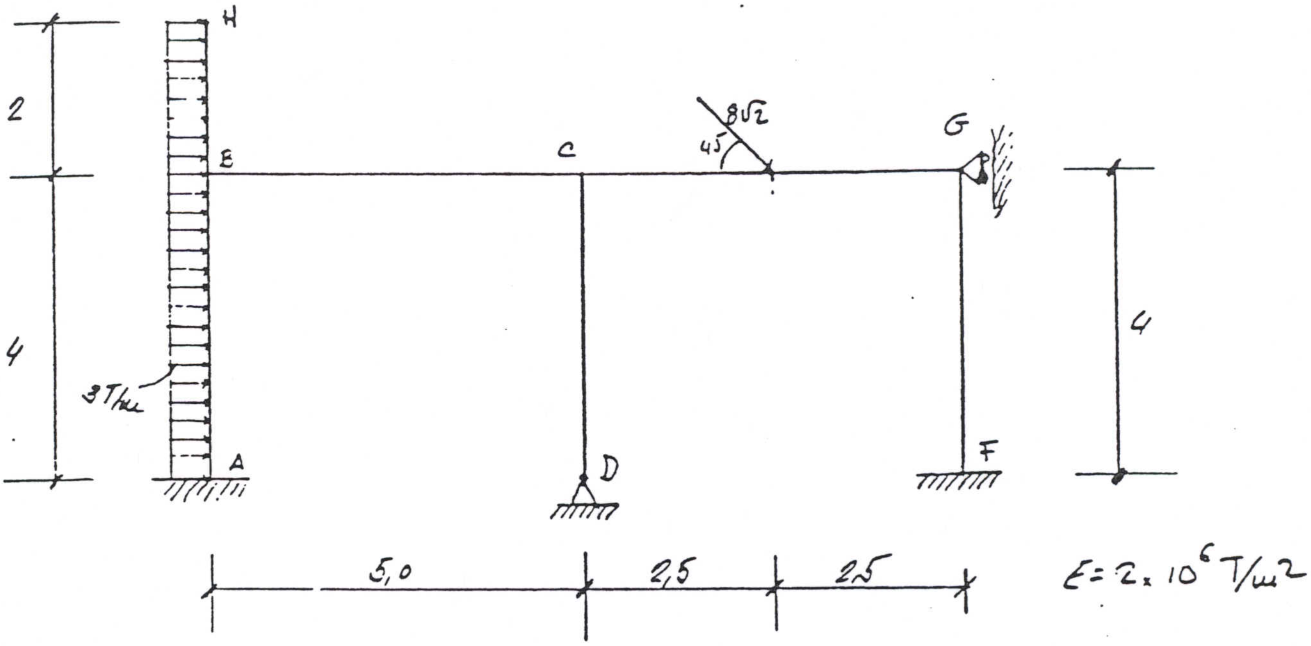
Las vigas AB, BH, CD, FG son de sección rectangular de 0,3x0,4 m. (ancho x alto) ; la viga CG, es también de sección rectangular de 0,3x0,5 m (ancho x alto).

La viga BC es de sección variable y de ella sabemos que: el momento de empotramiento en el nudo C al dar un momento M=4 en el extremo B es de MC=1,6 y el giro $\theta_B = 5 \times 10^{-4}$; y el momento de empotramiento en el nudo B MB es de 4 cuando aplicamos un momento M=5 en el extremo C.

~~Todas las longitudes están en metros~~

$E = 2 \times 10^6\ T/m^2$





$E = 2 \times 10^6\ T/m^2$

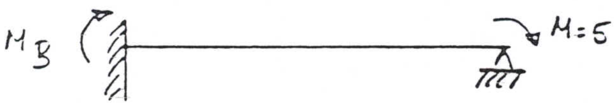
AB, BH, CD, FG $\Rightarrow 0{,}4 \times 0{,}30 \Rightarrow I = \frac{1}{12}\, 30 \times 40^3 = 16{,}0 \times 10^4\ cm^4$

CG $\Rightarrow 0{,}5 \times 0{,}30 \Rightarrow I = \frac{1}{12}\, 30 \times 50^3 = 31{,}25 \times 10^4\ cm^4$

BC :

$\theta_B = 5 \times 10^{-4}$

$M_c = 1{,}6$



$M_B = 4$

$K_{BC} = \frac{M_{BC}}{\theta_B} = \frac{4}{5 \times 10^{-4}} = 0{,}8 \times 10^4$; $C_{BC} = \frac{M_c}{M} = \frac{1{,}6}{4} = 0{,}4$

$C_{CB} = \frac{M_B}{M} = \frac{4}{5} = 0{,}8$

$K_{CB} = \frac{K_{BC}}{C_{CB}}\, C_{BC} = \frac{0{,}8 \times 10^4 \times 0{,}4}{0{,}8} = 0{,}4 \times 10^4$

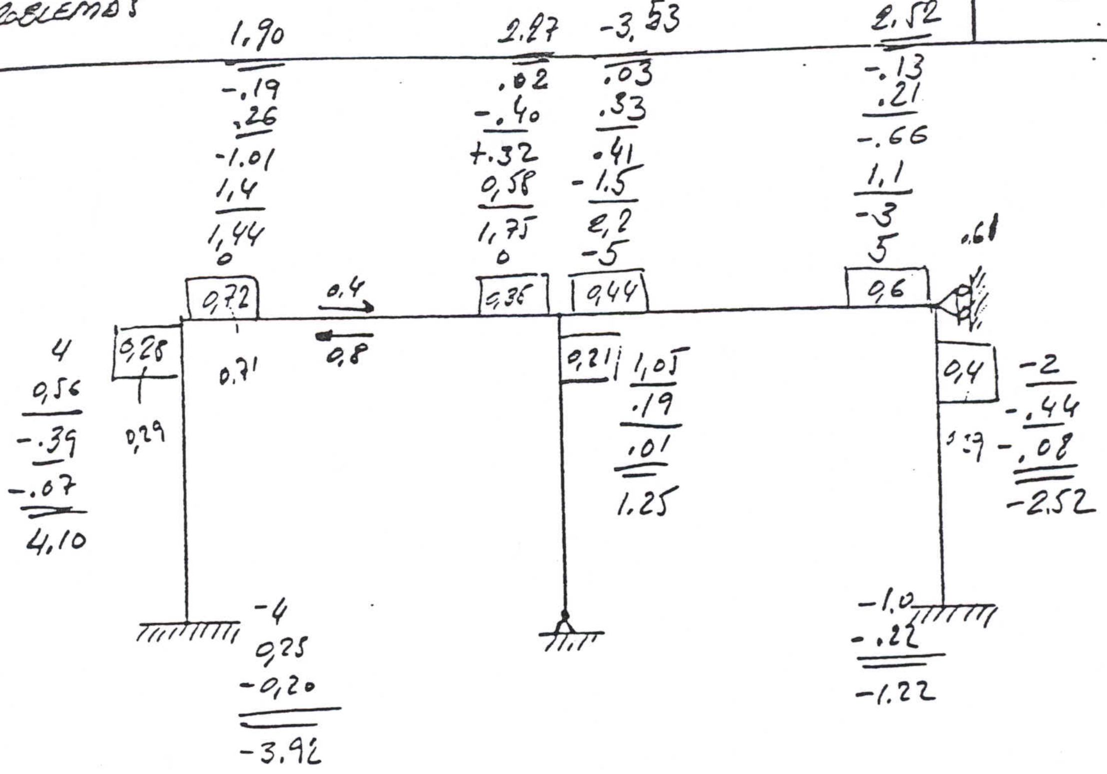
$K_{AB} = \frac{4EI}{4} = 32 \times 10^2 = K_{GF}$; $K_{CD} = \frac{3EI}{4} = 24 \times 10^2$; $K_{CG} = \frac{4EI}{5} = 50 \times 10^2$

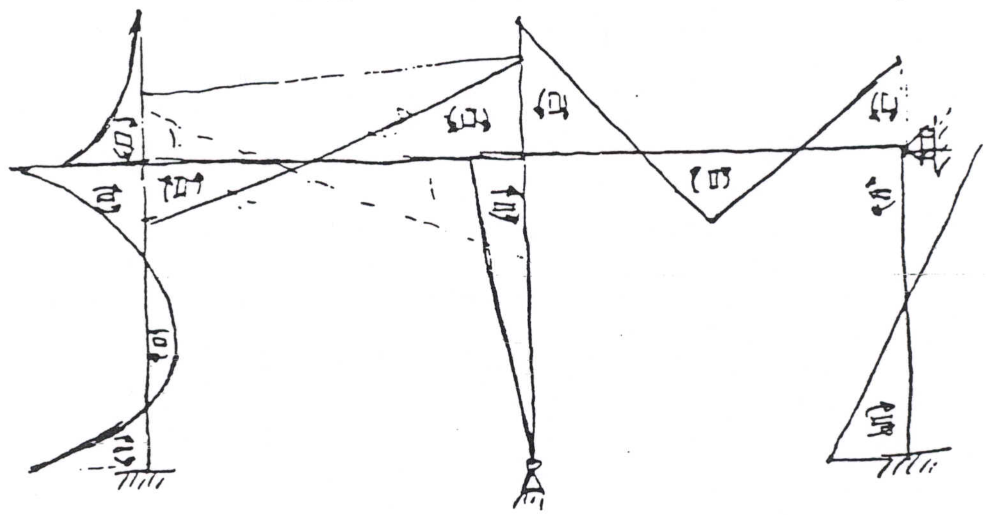
$M_{AB} = -\frac{PL^2}{12} = -\frac{3 \times 4^2}{12} = -4{,}0\ mT$; $M_{BA} = 4{,}0\ mT$

$M_{BH} = \frac{PL^2}{2} = \frac{3 \times 2^2}{2} = 6{,}0\ mT$

$M_{CG} = -\frac{PL}{8} = -\frac{8 \times 5}{8} = -5{,}0\ mT$; $M_{GC} = 5{,}0\ mT$

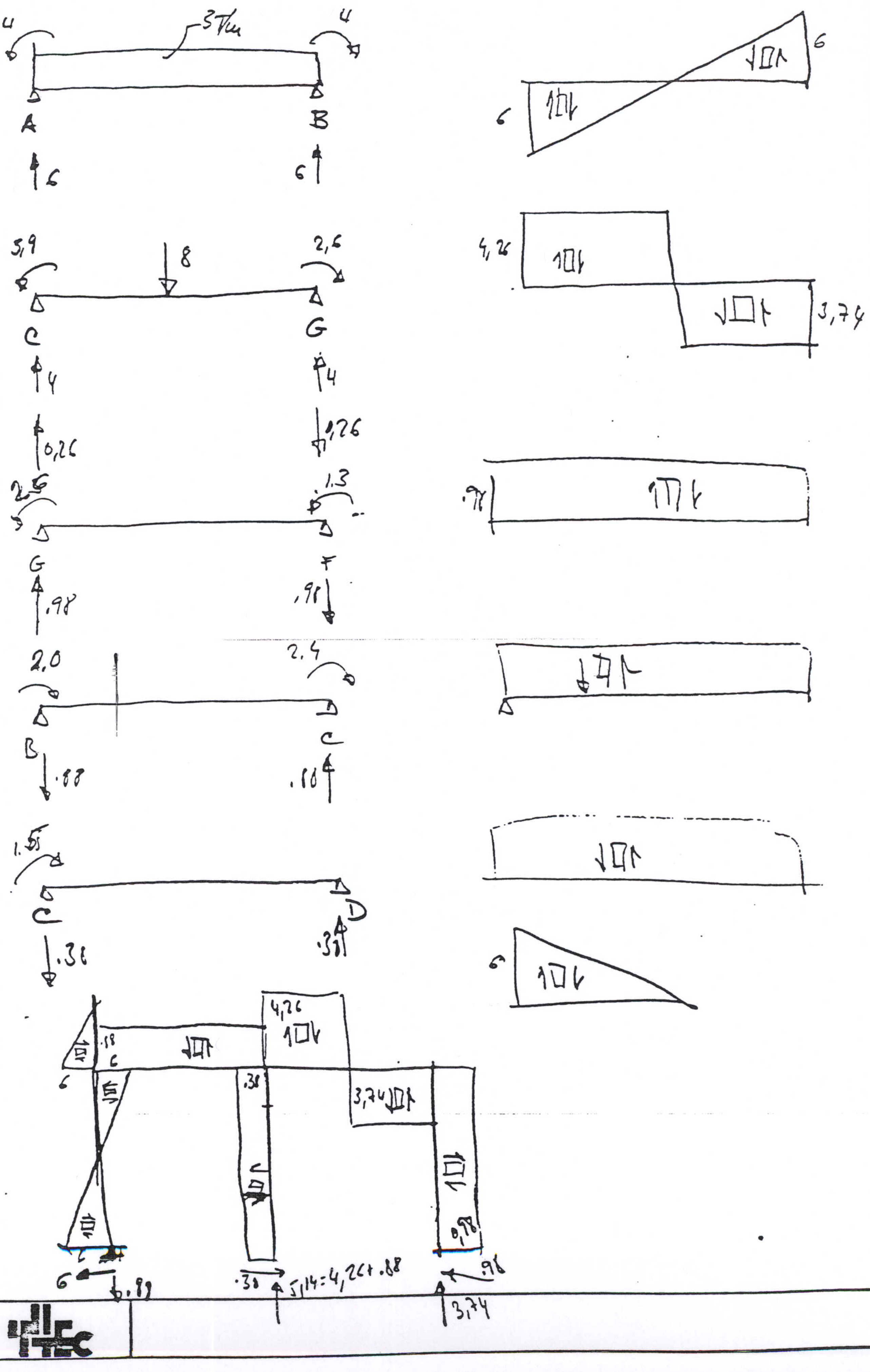
PROBLEMA 5

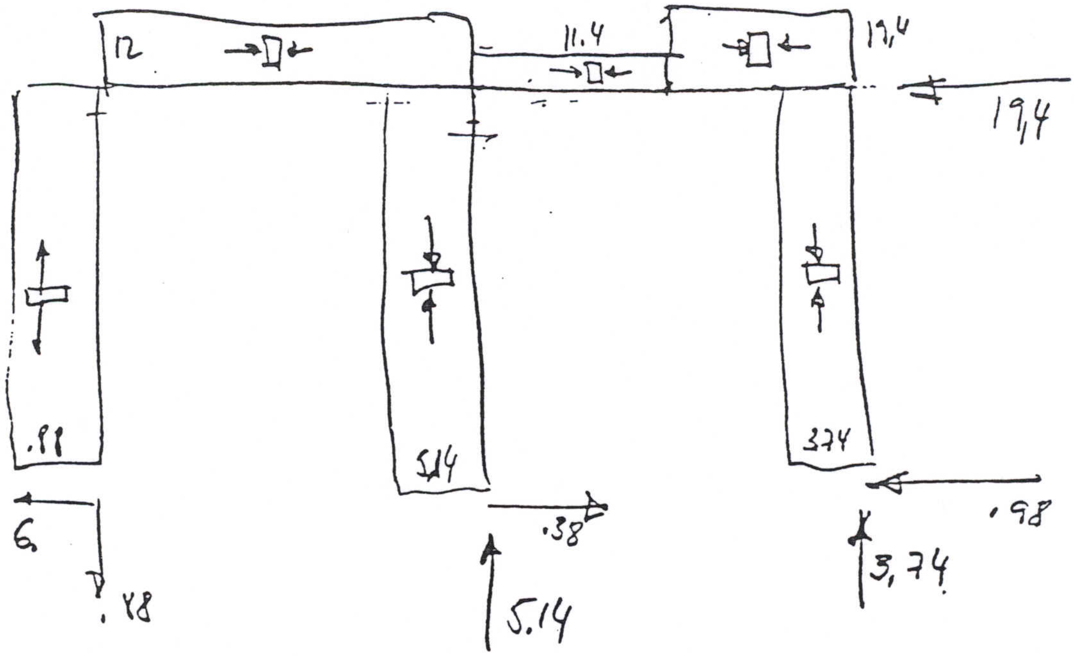

1,90
-,19
,26
-1.01
1,4
1,44
0
0,72
0,4
0,8
0,71
4
0,56
-.39
-.07
4,10
0,28
0,29
2.27
.02
-.40
+.32
0,58
1,75
0
0,36
-3,53
.03
.33
.41
-1.5
2,2
-5
0,44
0,21
1,05
.19
.01
1.25
2,52
-.13
.21
-.66
1,1
-3
5
0,6
.61
0,4
-2
-.44
-.08
-2,52
-4
0,28
-0,20
-3.92
-1,0
-.22
-1.22

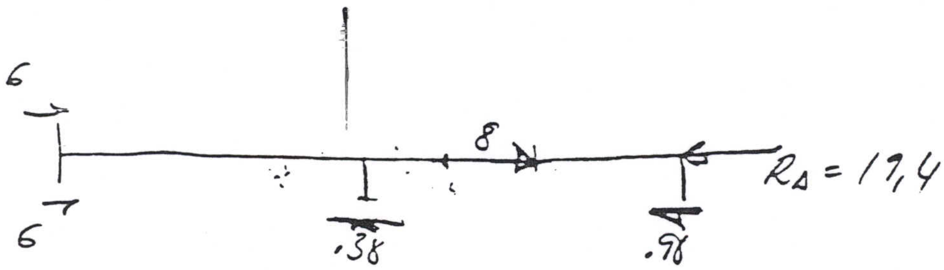


PROBLEMA J

REACCIONES | CORTANTES




12
11.4
19,4
19,4
.98
5.14
3.74
6
.38
.98
3,74
.48
5.14


6
8
RA = 19,4
6
.38
.98

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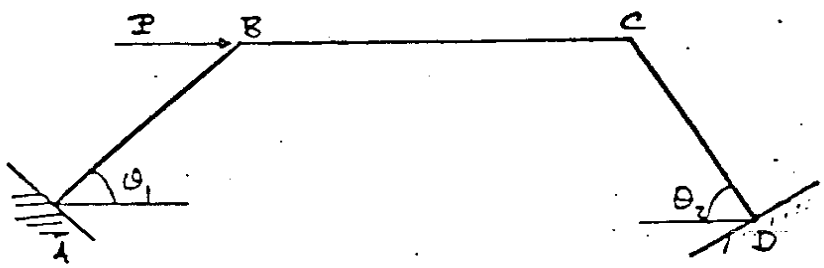
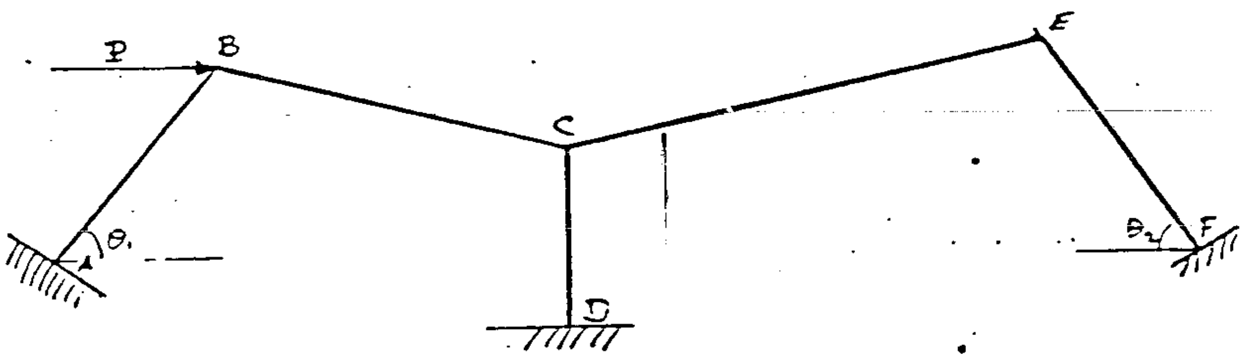
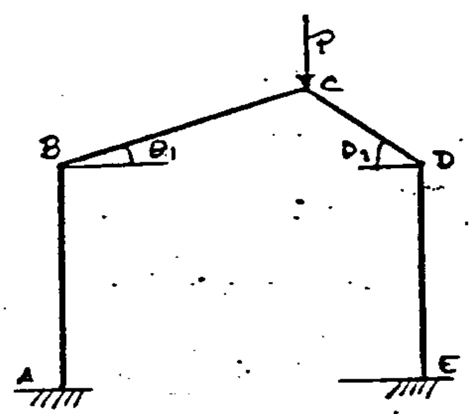
Asignatura: ANALISIS DE ESTRUCTURAS-METODOS NUMERICOS

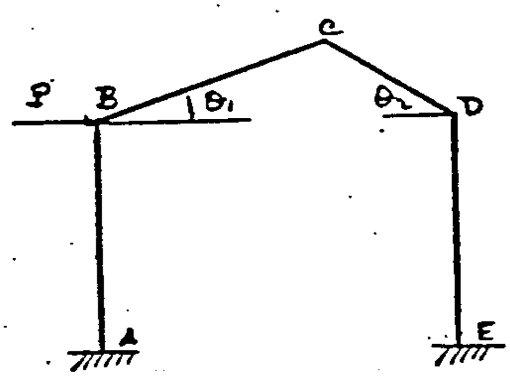
Problema 6:

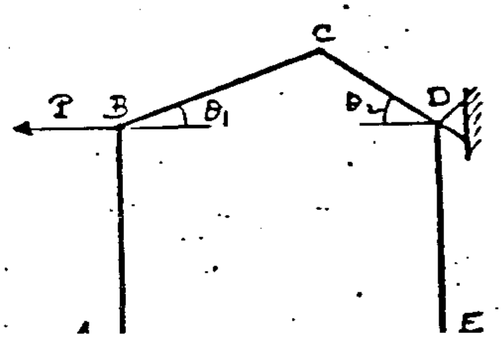
Dibujar las deformadas de las estructuras que se indican a continuación, indicando su grado de traslacionalidad y determinando claramente los desplazamientos de sus nudos.

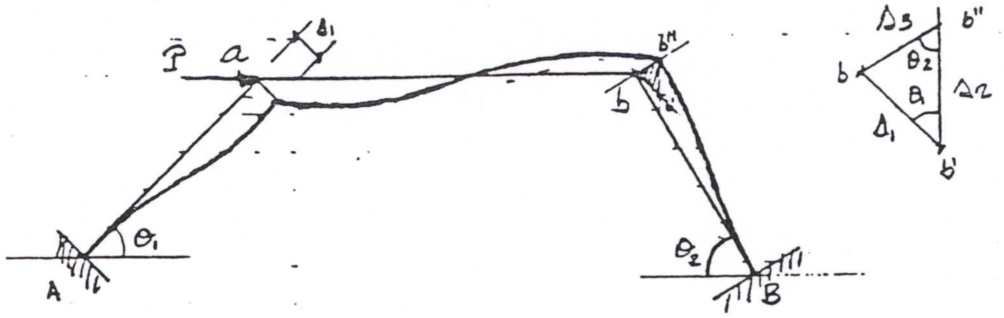


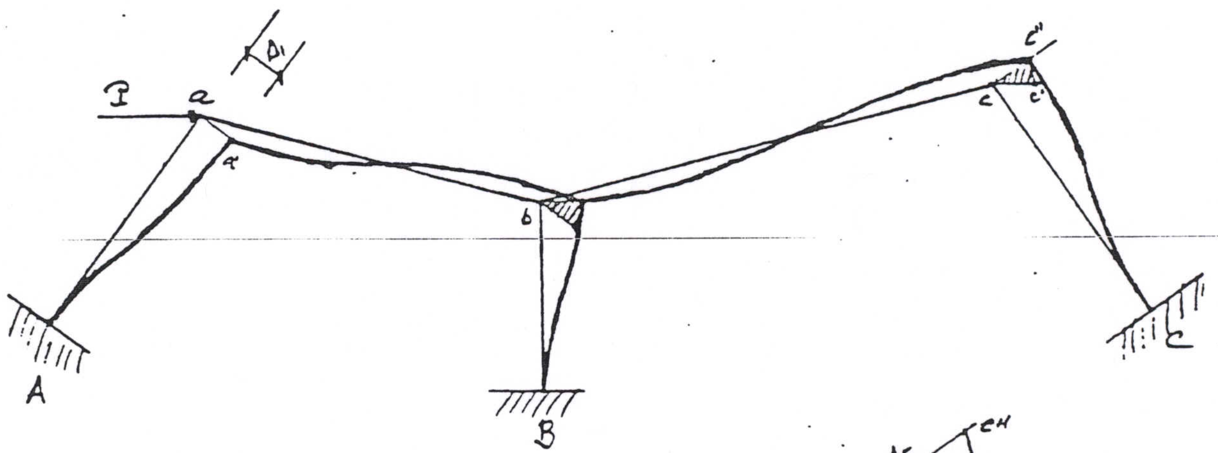


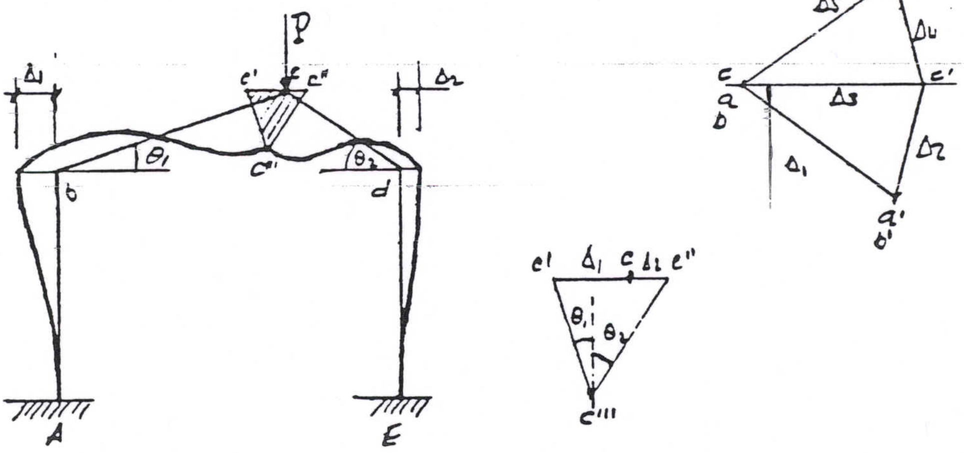


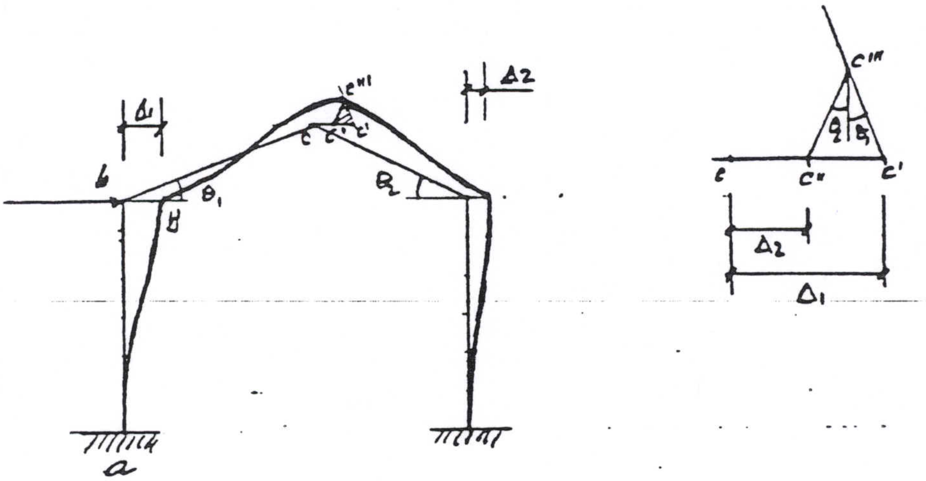


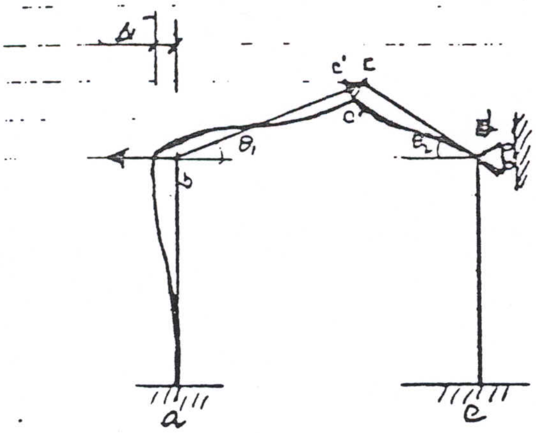


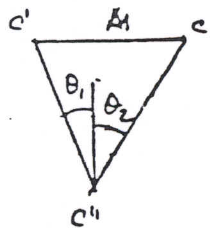

P
a
Δ1
b''
b
θ1
θ2
A
B
Δ3
Δ2
Δ1
b'


P
a
Δ1
b
c''
c
c'
A
B
C


P
Δ1
Δ2
c'
c''
c'''
θ1
θ2
b
d
A
E
Δ5
Δ4
Δ3
Δ2
Δ1
a'
b'


Δ1
Δ2
b
c'''
θ1
θ2
a
c''
c'
Δ2
Δ1


Δ
c' c
c''
θ1
θ2
d
b
a
e


c'
Δ1
c
θ1
θ2
c''

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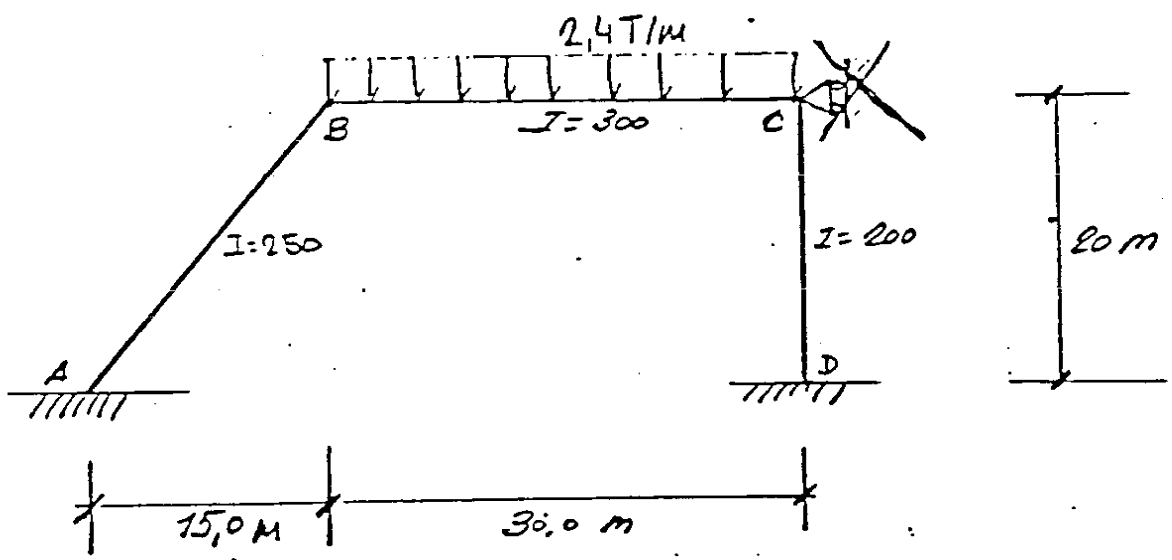
Asignatura: <u>ANALISIS DE ESTRUCTURAS -METODOS NUMERICOS</u>

Problema 7:

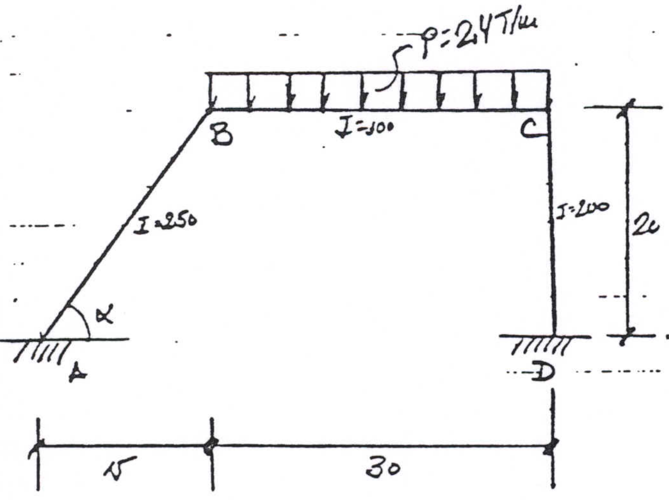
Resolver el problema nº 3, pero quitando el apoyo del nudo C

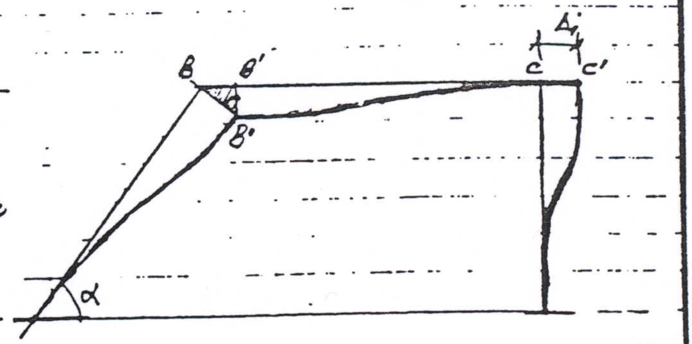
NOTA: A continuación se repite el enunciado del citado problema nº6

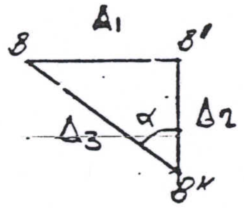
En la estructura intraslacional de la figura, determinar las leyes de momentos flectores, esfuerzos constantes, axiles, así como las reacciones.



Del problema Nº 6, tenemos que $R_c = \overleftarrow{27}$







$$\frac{\Delta_1}{\text{sen}\,\alpha} = \frac{\Delta_2}{\cos\alpha} = \frac{\Delta_3}{1} \qquad (\text{sen}\,\alpha = 0{,}8;\ \cos\alpha = 0{,}6)$$

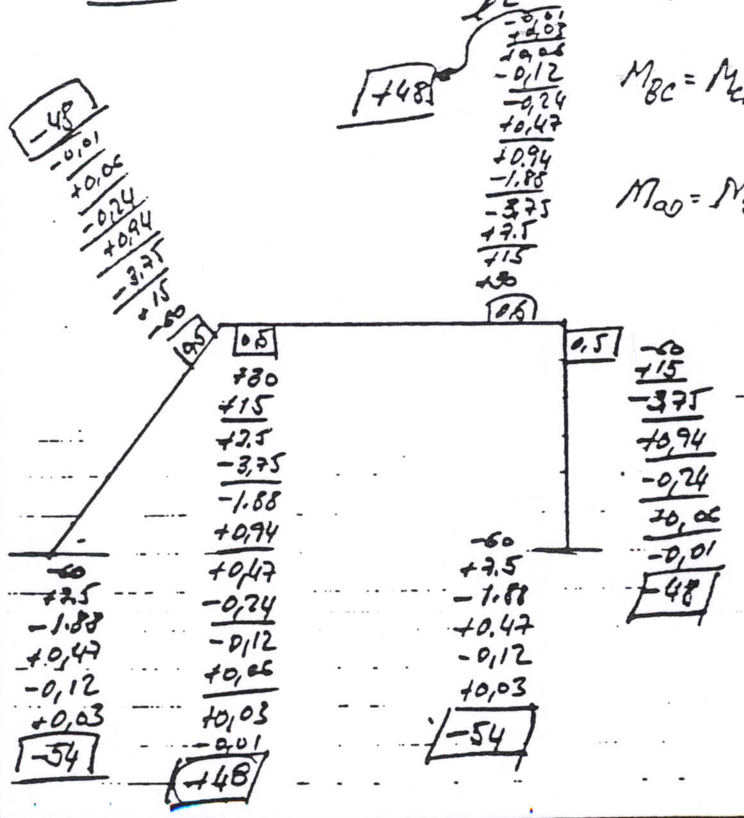
$$\Delta_1 = \Delta_2 \frac{0{,}8}{0{,}6} = \Delta_2 \frac{20}{15}$$

Despl. relativos:
- $\Delta_1 = 20$
- $\Delta_2 = 15$
- $\Delta_3 = 25$

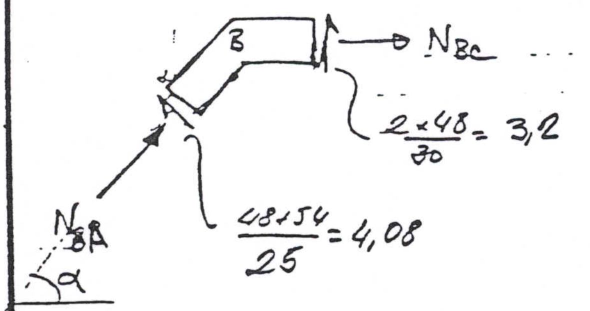
Momentos: $M = \frac{6EI\Delta}{\ell^2} \Rightarrow M_{BA} = M_{AB} = -\frac{6 \times E \times 250 \times 25}{25^2} = -60\,E$

$$M_{BC} = M_{CB} = \frac{6 \times E \times 300 \times 15}{30^2} = 30\,E$$

$$M_{CD} = M_{DC} = -\frac{6 \times E \times 200 \times 20}{20^2} = -60\,E$$



Equilibrio del nudo B: (Fuerzas verticales)

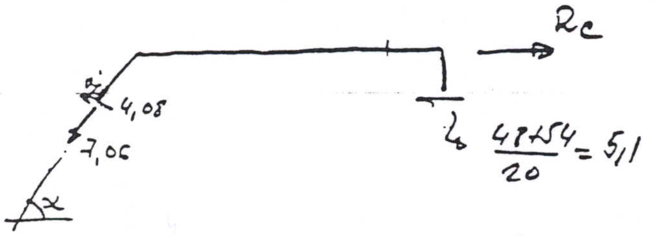


$$N_{BA}\,\text{sen}\,\alpha + 4,08\cos\alpha + 3,2 = 0$$

$$N_{BA} = -\frac{4,08\times 0,6 + 3,2}{0,8} = -7,06$$

Es decir que tiene sentido opuesto al indicado (barra a tracción.)

Obtención de la reacción.

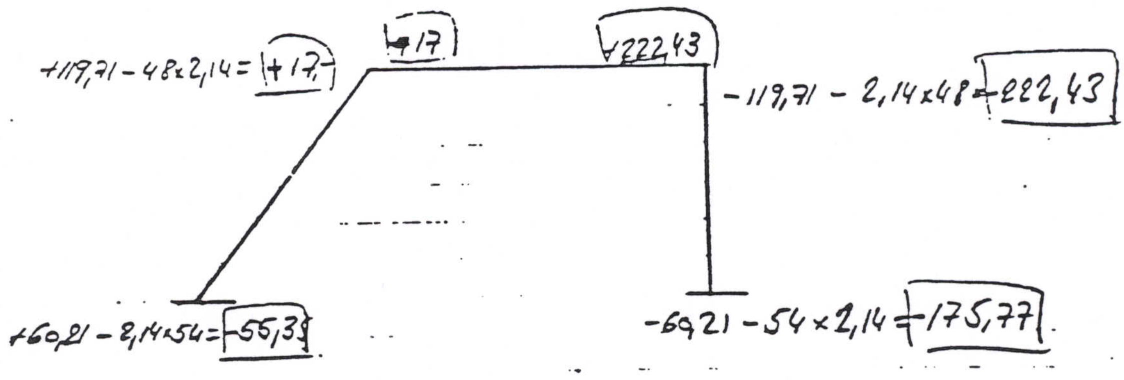


$$R_c = 7,06\cos\alpha + 4,08\,\text{sen}\,\alpha + 5,1 =$$
$$= 7,06\times 0,6 + 4,08\times 0,8 + 5,1 =$$
$$= 12,6$$

Equilibrio:

$$27 - 12,6\,X = 0 \Rightarrow X = \frac{27}{12,6} = 2,14$$

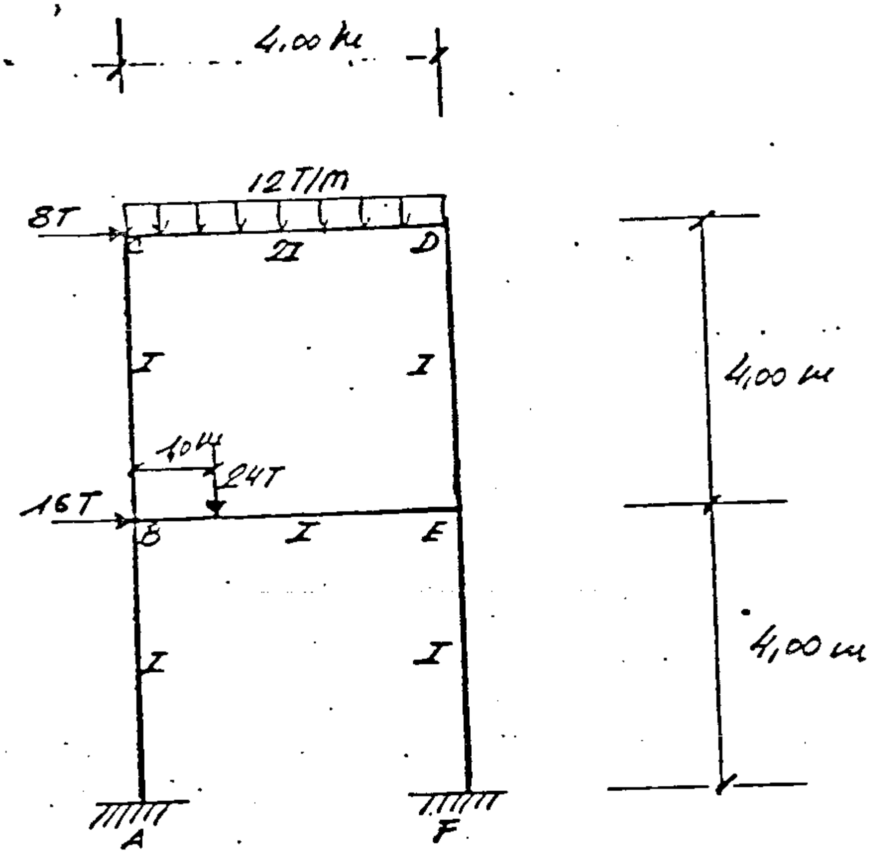
RESULTADOS FINALES:

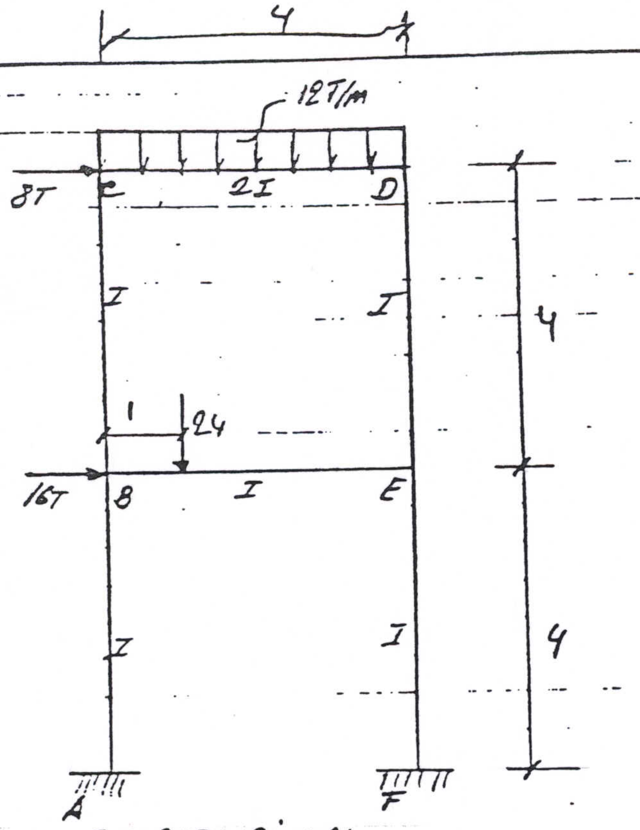


Asignatura: ANALISIS DE ESTRUCTURAS -METODOS NUMERICOS

Problema 8 :

Resolver la estructura traslacional de la figura que se indica a continuación ,





$2 \times n - b - r = 12 - 6 - 4 = 2$

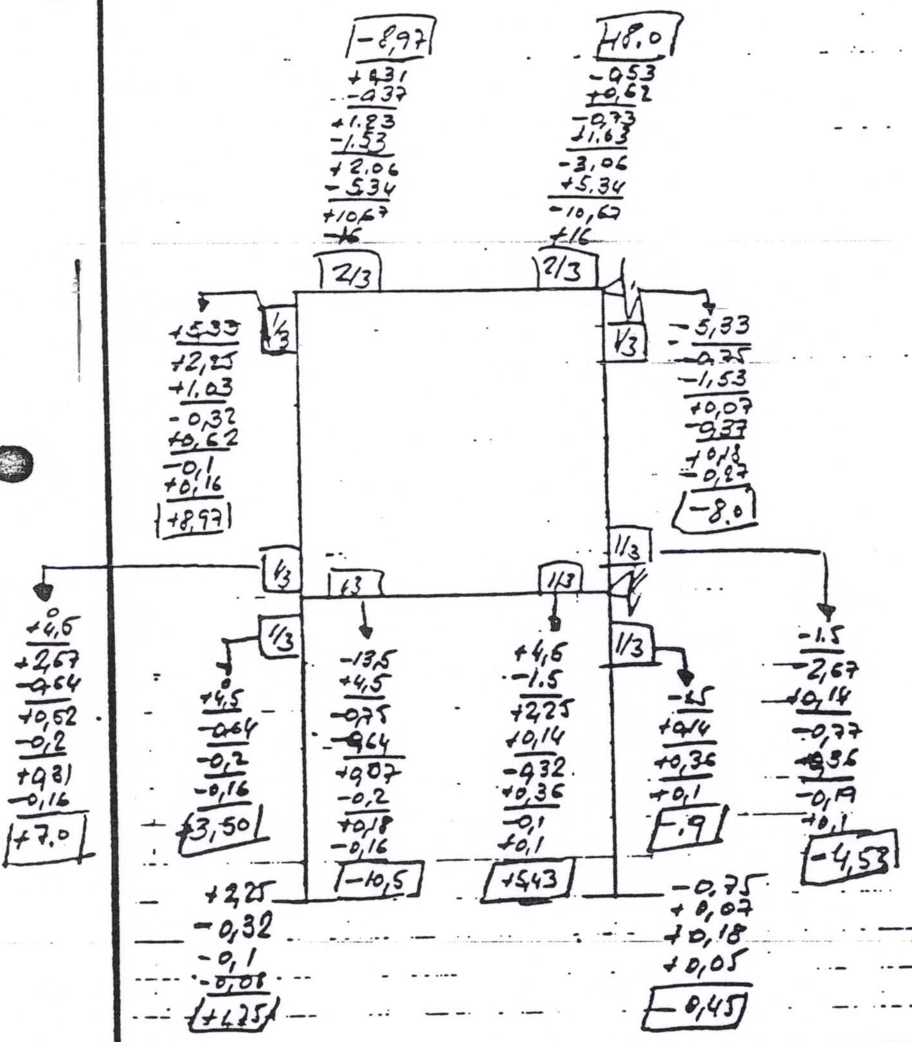
Traslacional de grado dos.

$$M_{BE} = -\frac{24 \times 1 \times 3^2}{4^2} = -13{,}5$$

$$M_{EB} = \frac{24 \times 3 \times 1^2}{4^2} = 4{,}5$$

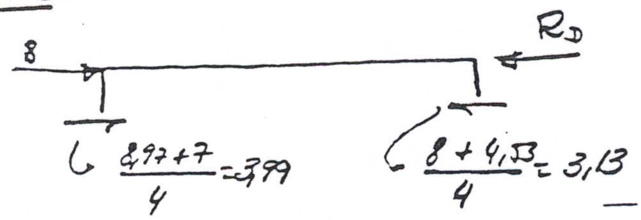
$$-M_{CD} = M_{DC} = \frac{12 \times 4^2}{12} = 16$$

① Cross intraslacional

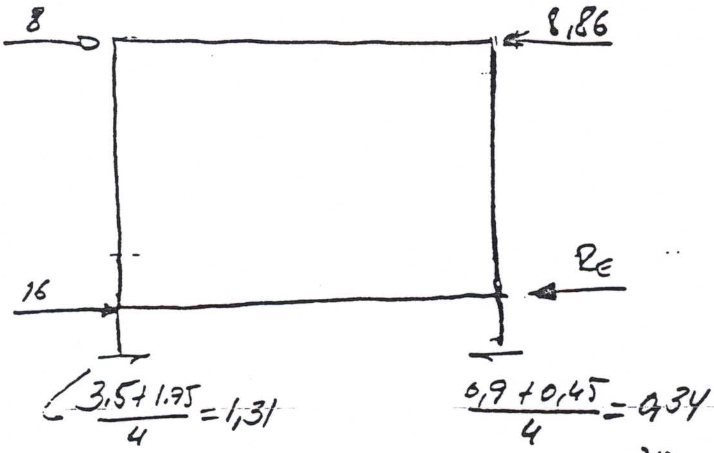


170-190

Reacciones:

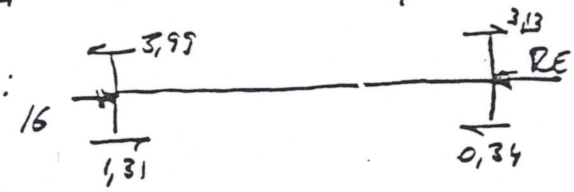


$$R_D = 8 + 3,99 - 3,13 = \underline{8,86}$$



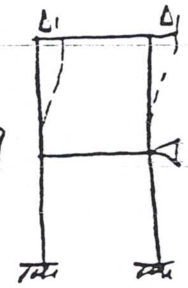
$$R_E = 16 + 8 + 1,31 - 0,34 - 8,86 =$$
$$= 16,11$$

o también:



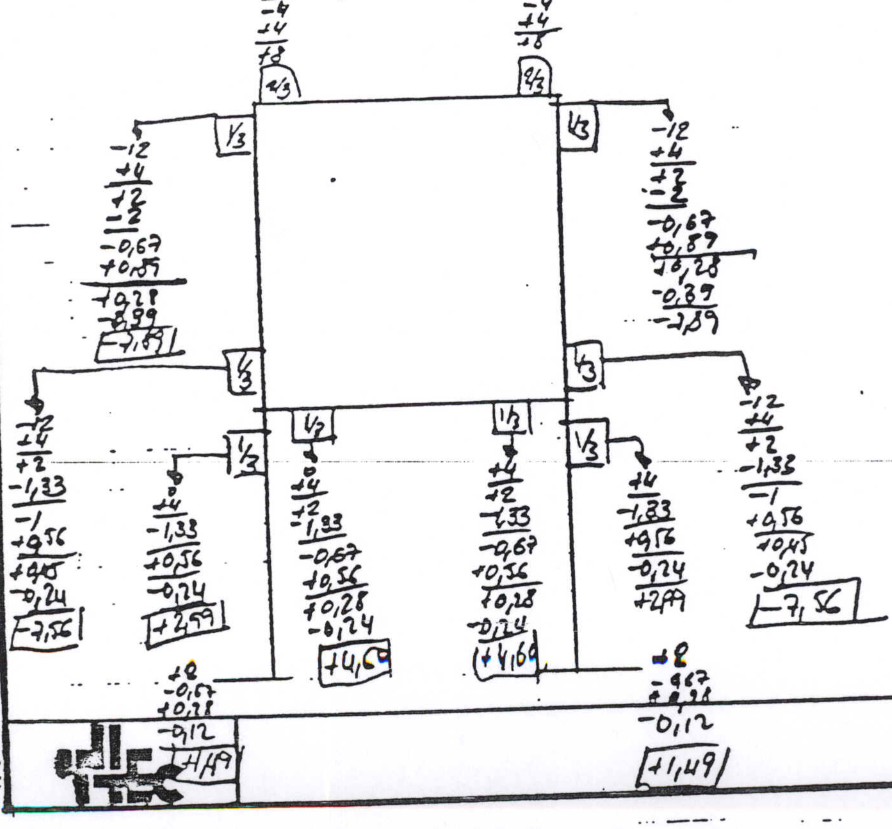
$$R_E = 16 + 1,31 + 3,13 - 0,34 - 3,99 =$$
$$= 16,11$$

1er TRASLACIONAL

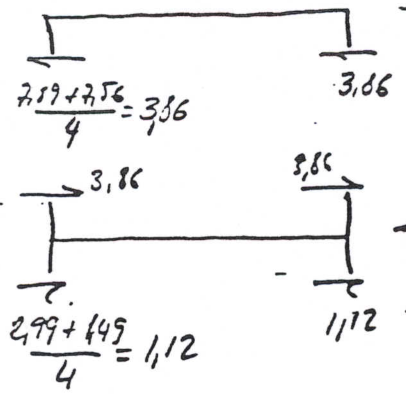


$$M = \frac{6EI\Delta_1}{l^2} = \frac{6}{16} EI\Delta_1 = \frac{3}{8} EI\Delta_1$$

haciendo: $EI\Delta_1 = 32 \Rightarrow M = 12$



Reacciones:

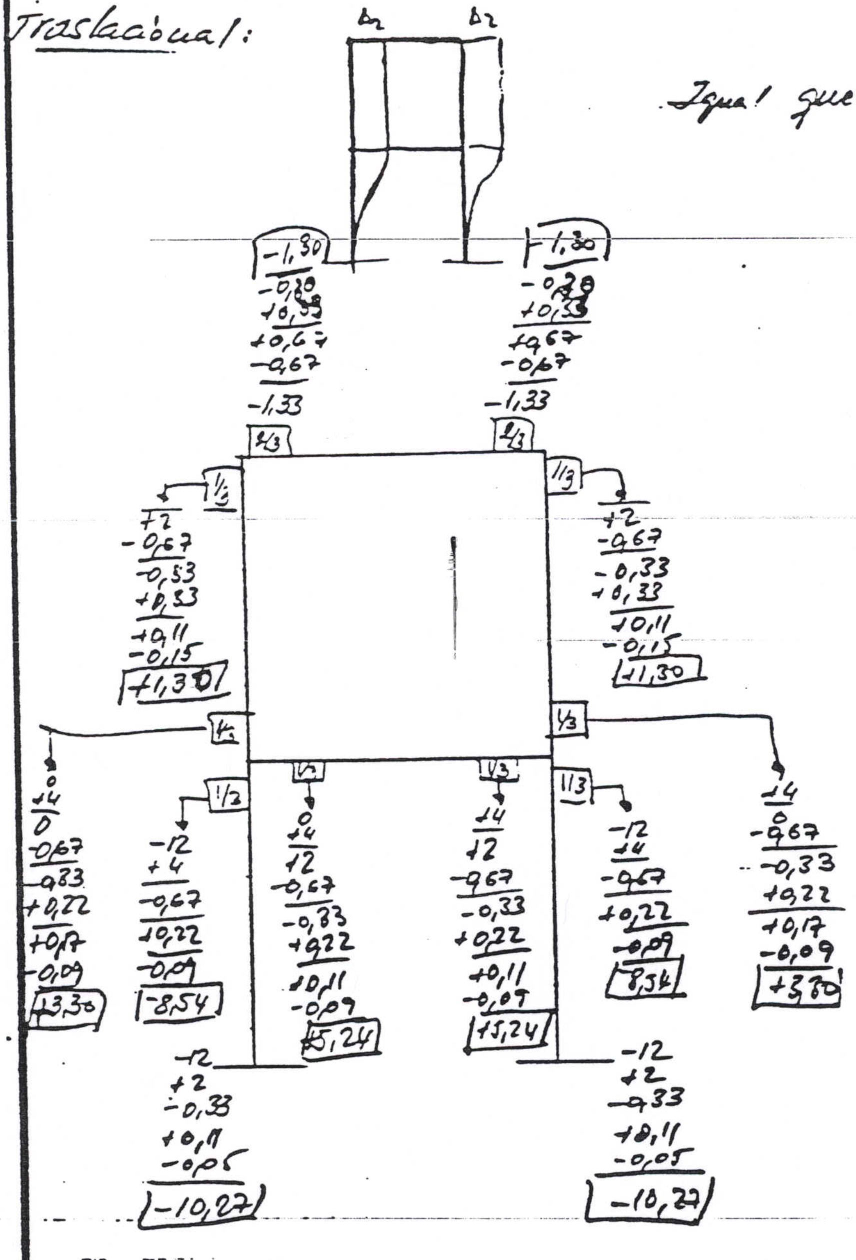


→ $R_D = 7,73$

← $R_E = 9,96$

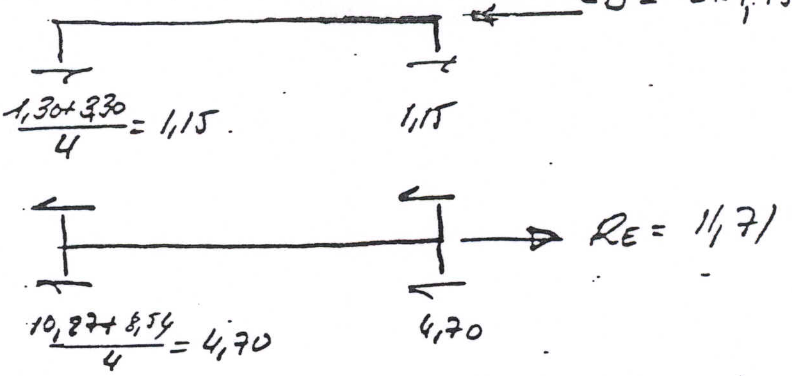
② Traslacional:

Igual que antes: M=12



Reacciones:

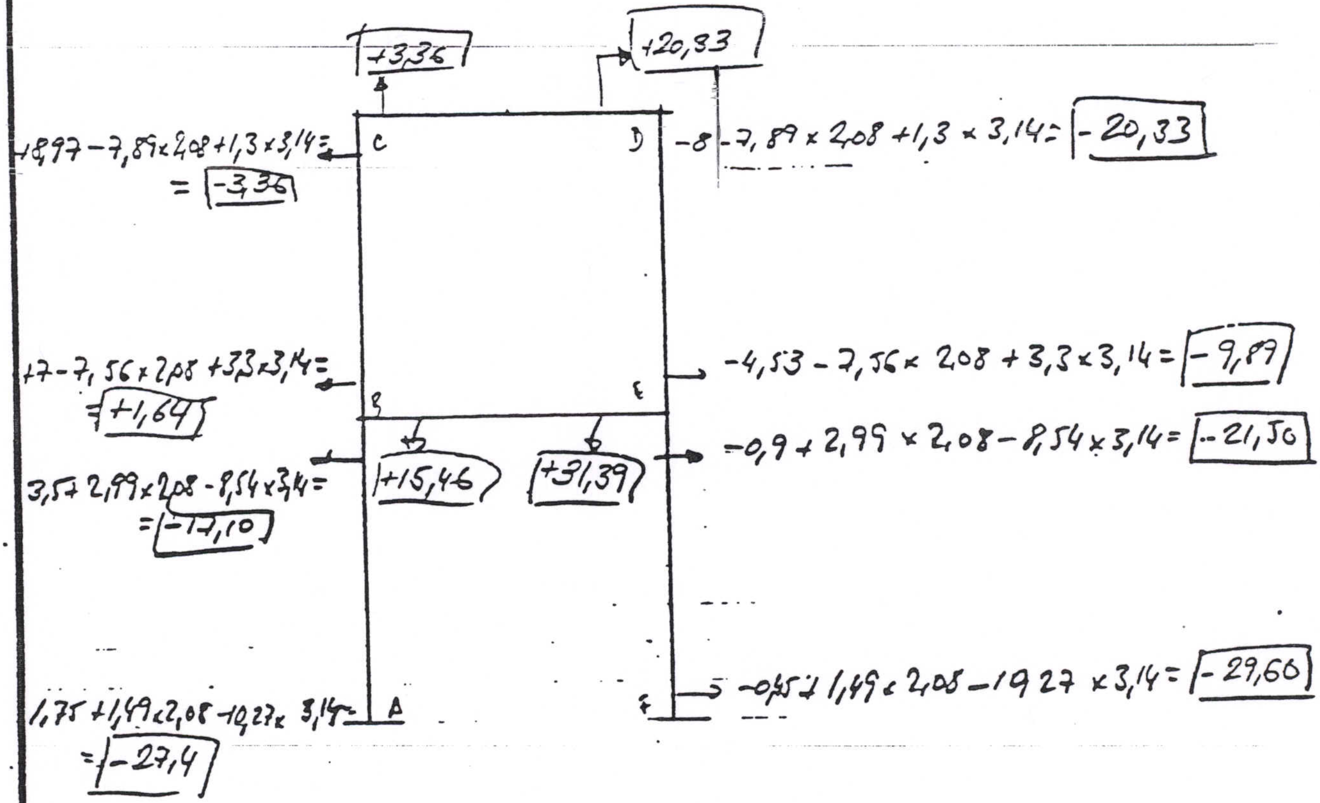
$R_D = 2 \times 1{,}15 = 2{,}30$



Equilibrio:

$$\left.\begin{aligned} 8{,}86 - 7{,}73\,X + 2{,}3\,Y &= 0 \\ 16{,}11 + 9{,}96\,X - 11{,}71\,Y &= 0 \end{aligned}\right\}$$

$$\left.\begin{aligned} 7{,}73\,X - 2{,}3\,Y &= 8{,}86 \\ -9{,}96\,X + 11{,}71\,Y &= 16{,}11 \end{aligned}\right\} \Rightarrow \boxed{X = 2{,}08 \quad Y = 3{,}14}$$



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Asignatura: ANALISIS DE ESTRUCTURAS-METODOS NUMERICOS

Problema 10: (Simetría y antimetría)

Dado el marco de la figura, se solicita:

a) Dibujar las leyes de esfuerzos; flectores, cortantes y axiles

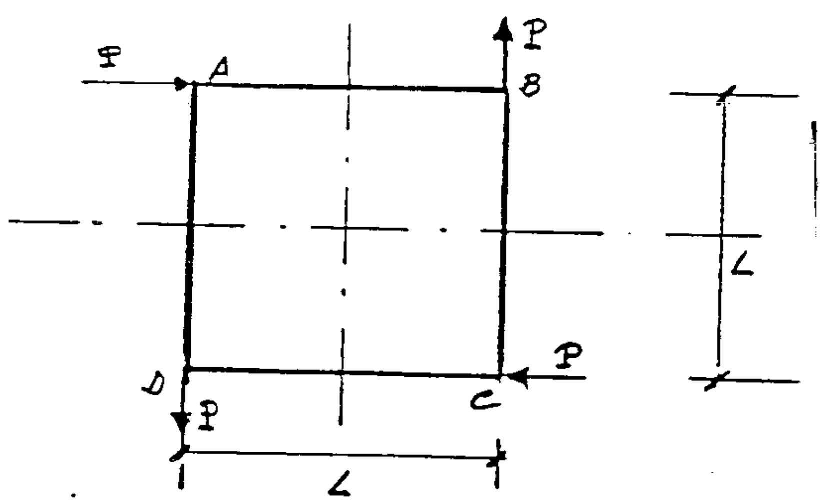
b) Dibujar la deformada de la estructura acotando sus valores característicos

c) Calcular las longitudes finales de las diagonales AC y BD

Características: Módulo de elasticidad constante E

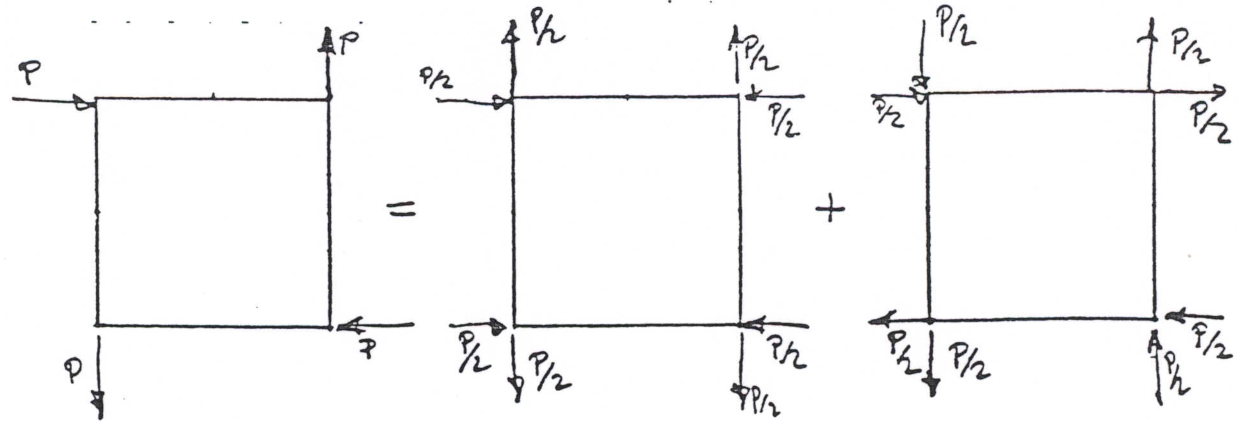
Sección constante A

Momento de inercia constante I



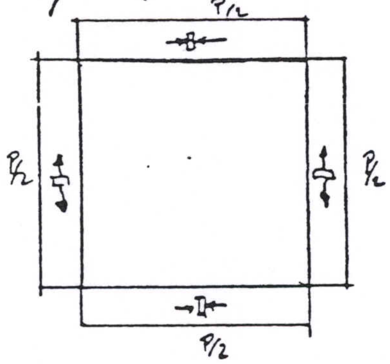
PROBLEMA: 31

1

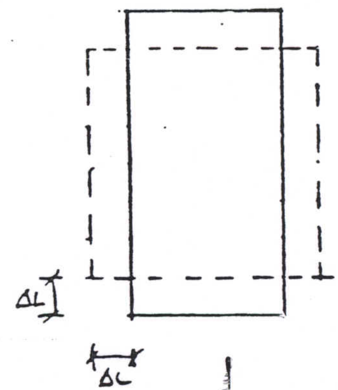


ESTADO 1 + ESTADO 2

ESTADO 1: Solo hay esfuerzos axiles.

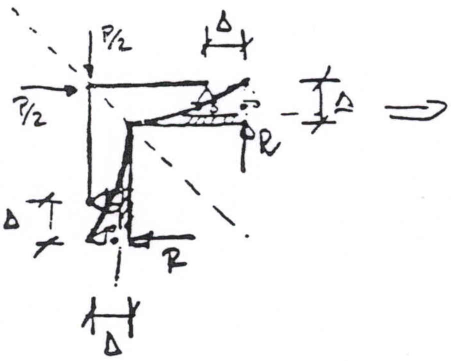


Deformada:

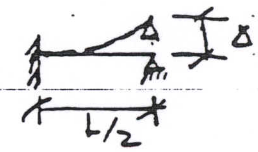


$$\Delta L = \frac{NL}{AE} = \frac{P}{2}\,\frac{L}{EA}$$

ESTADO 2: Estructura antimétrica.



Como el nudo central no gira..., luego el problema se reduce al de una viga empotrada apoyada, con un descenso Δ en el apoyo.

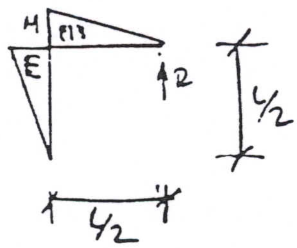


$$\alpha' = \frac{M\,(L/2)}{3EI} \qquad \alpha'' = \frac{\Delta}{(L/2)}$$

Por compatibilidad: $\alpha' + \alpha'' = 0$

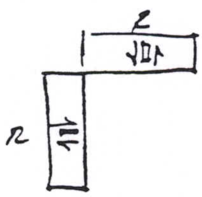
$$\frac{ML}{6EI} = -\frac{2\Delta}{L} \Rightarrow \boxed{\Delta = \frac{ML^2}{12EI}}$$

Momento flector: $M = R\frac{L}{2} = \frac{P}{2}\frac{L}{2} = \frac{PL}{4}$



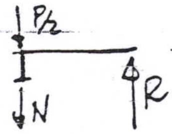
Por tanto $\boxed{\Delta = \frac{PL}{4}\frac{L^2}{12EI} = \frac{PL^3}{48EI}}$

Cortantes:



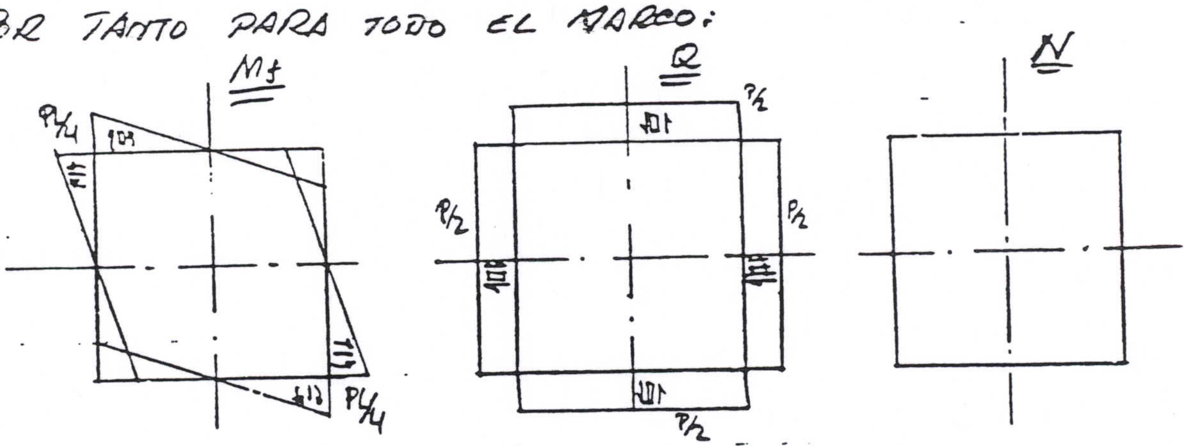
Siendo $R = \frac{P}{2}$

Axiles:



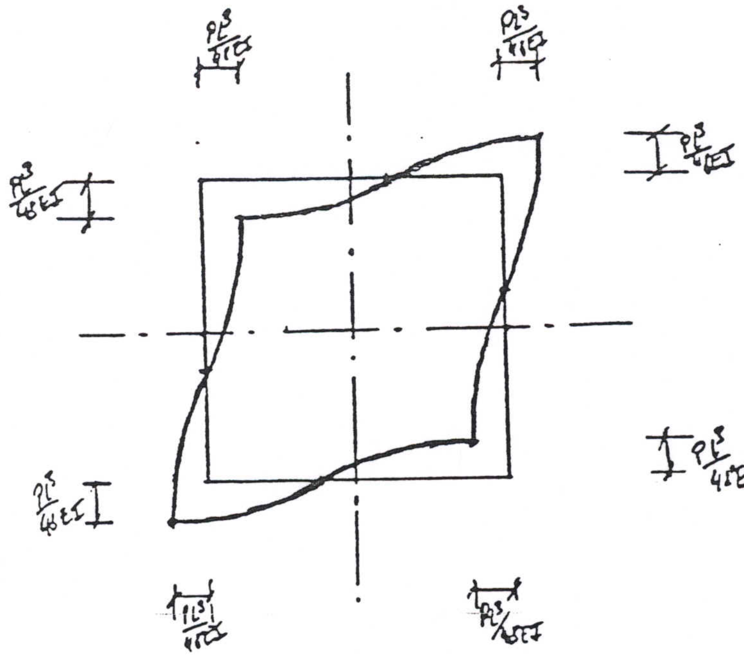
$N + \frac{P}{2} = R \Rightarrow \boxed{N = 0}$

POR TANTO PARA TODO EL MARCO:

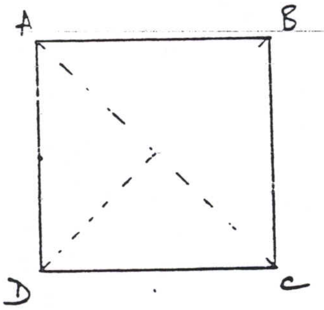


POR TANTO ESTADO 1 + ESTADO 2 ⇒
- Mf → Es el del ESTADO 2
- Q → Es el del ESTADO 2
- N → Es el del ESTADO 1

Movimientos en el ESTADO 2: (caso antimétrico).



MOVIMIENTOS TOTALES: Se obtienen sumando algebraicamente los obtenidos para cada estado.



DISTANCIA BD:

Longitud inicial: $L\sqrt{2}$

(+) Estado 1 (simétrico): $\frac{PL}{2EA}\sqrt{2} \times 2$

(+) Estado 2 (antimétrico): $\frac{PL^3}{48EI}\sqrt{2} \times 2$

$$\overline{BD}_{final} = \left[L + \frac{PL}{EA} + \frac{PL^3}{24EI}\right]\sqrt{2}$$

DISTANCIA AC:

Longitud inicial: $L\sqrt{2}$

(+) Estado 1 (simétrico): $\frac{PL}{2EA}\sqrt{2} \times 2$

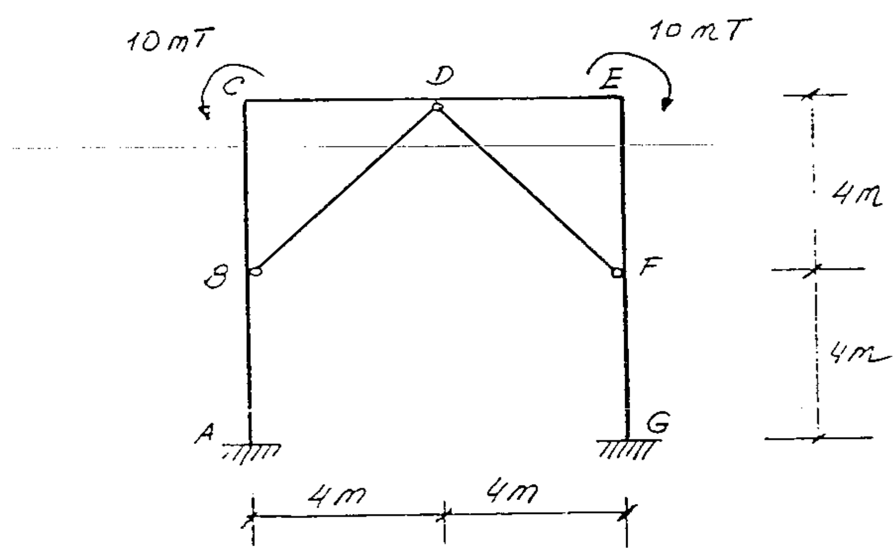
(−) Estado 2 (antimétrico): $\frac{PL^3}{48EI}\sqrt{2} \times 2$

$$\overline{AC}_{final} = \left[L + \frac{PL}{AE} - \frac{PL^3}{24EI}\right]\sqrt{2}$$

UNIVERSIDAD NACIONAL DE EDUCACION A DISTANCIA

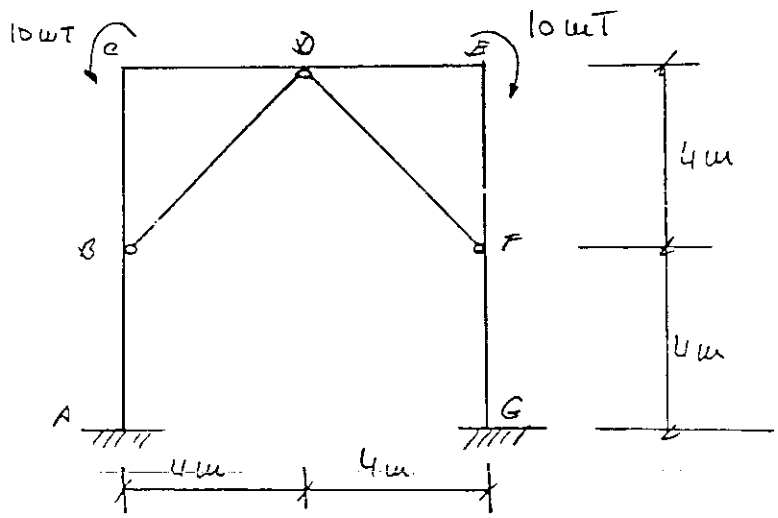
Asignatura: ANALISIS DE ESTRUCTURAS-METODOS NUMERICOS

PROBLEMA 11.- Calcular el giro en el nudo E de la estructura simetrica representada en la figura, si se supone que todas las barras son inextensibles y con E*I= 4000. T*m²



PROBLEMA:

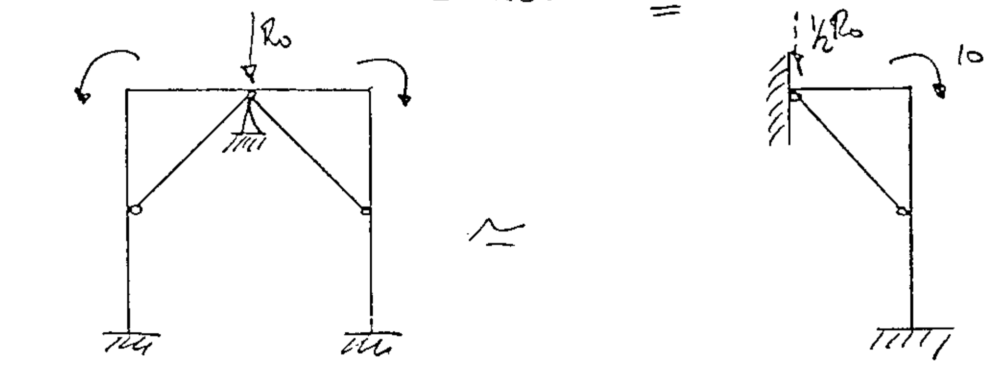
Calcular el giro en el nudo E de la estructura simétrica representada en la figura si se supone que todas las barras son inextensibles y con $EI = 6000\ m^2T$.



Traslacional de grado 1, será de grado 2 si las cargas no fueran simétricas.

Si se levanta D una cantidad δ como C y E no se mueven por simetría los nudos B y F se desplazan horizontalmente la misma cantidad δ de forma simétrica.

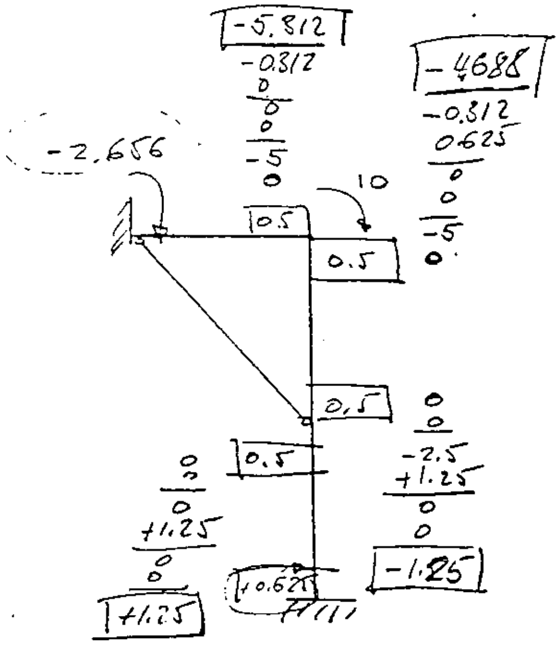
ESTADO 0 : INTRASLACIONAL IMPIDIENDO EL MOVIMIENTO VERTICAL DEL NUDO D.

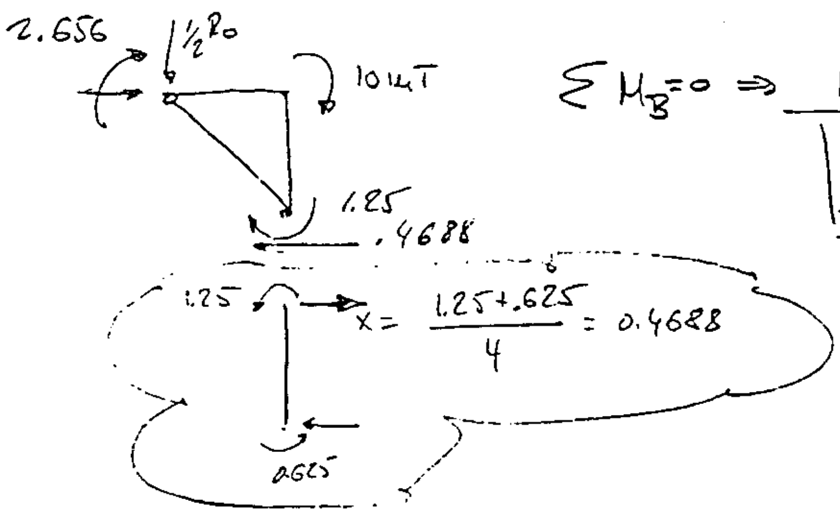


PARA TODAS LAS BARRAS:

$$\frac{4EI}{L} = \frac{4 \times 6000}{4} = 6000 \Rightarrow \text{Coef. de reparto para todas las barras } 0.5$$

CROSS :

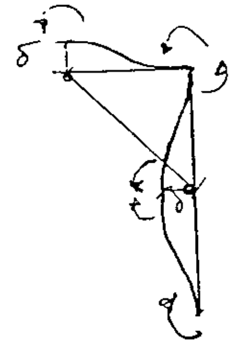




$$\sum M_B = 0 \Rightarrow 10 + 2,656 + 1.25 + .4688 \times 4 = \frac{R_0}{2} 4$$

$$R_0 = 7.8906 \text{ T}$$

ESTADO I: TRASLACIONAL

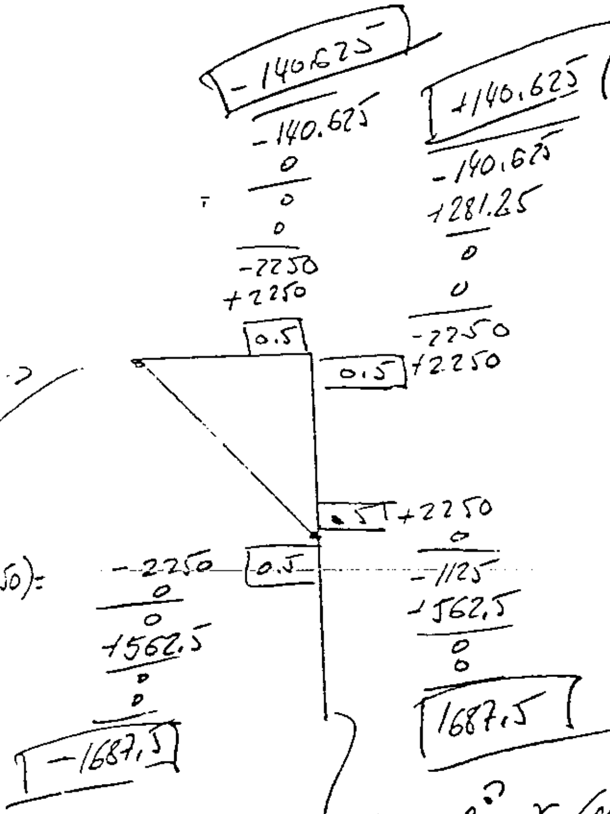


$$M\Big|_{\delta=1} = \frac{6EI\delta}{L^2} = \frac{6 * 6000 * 1}{16} = 2250.\ \text{mT}$$

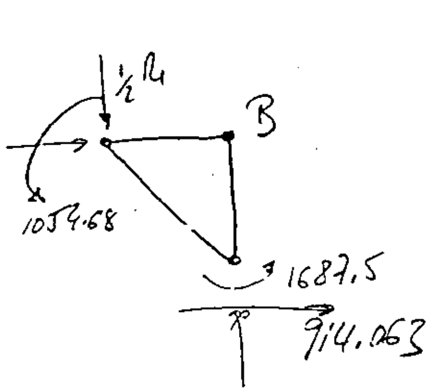
CROSS:

$$\theta_1 = \frac{(M_1 - M_1^{\circ}) - \gamma_1 (M_2 - M_2^{\circ})}{K_1 (1 - \gamma_1 \gamma_2)}$$

$$M_1 = M_1^{\circ} + \gamma_1 (M_2 - M_2^{\circ}) =$$
$$= 2250 + \frac{1}{2}(-140.625 - 2250) =$$
$$= 1054.68$$

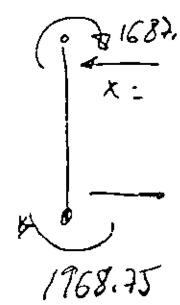


$$M_1 = M_1^{\circ} + \gamma_1 (M_2 - M_2^{\circ}) =$$
$$= -2250 + \frac{1}{2}\left(-1687.5 - (2250)\right) = -1968.75$$



$$\sum M_B = 1054.68 + 1687.5 + 4 * 914.063 + \frac{R_1}{2} 4$$

$$R_1 = -3199.215$$



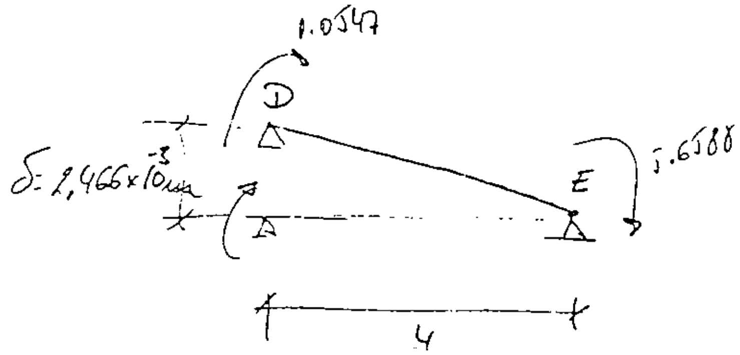
$$x = \frac{1687.5 + 1968.75}{4} = 914.063$$

$$R_0 + \delta R_1 = 0$$

$$7.8906 + \delta(-3199.215) = 0$$

$$\delta = 2.466 * 10^{-3} \text{ m.}$$

GIRO EN E:



$$M_D = -2,656 + 2,466 * 10^{-3} * 1054.68 = -0,0547 \text{ mT}$$

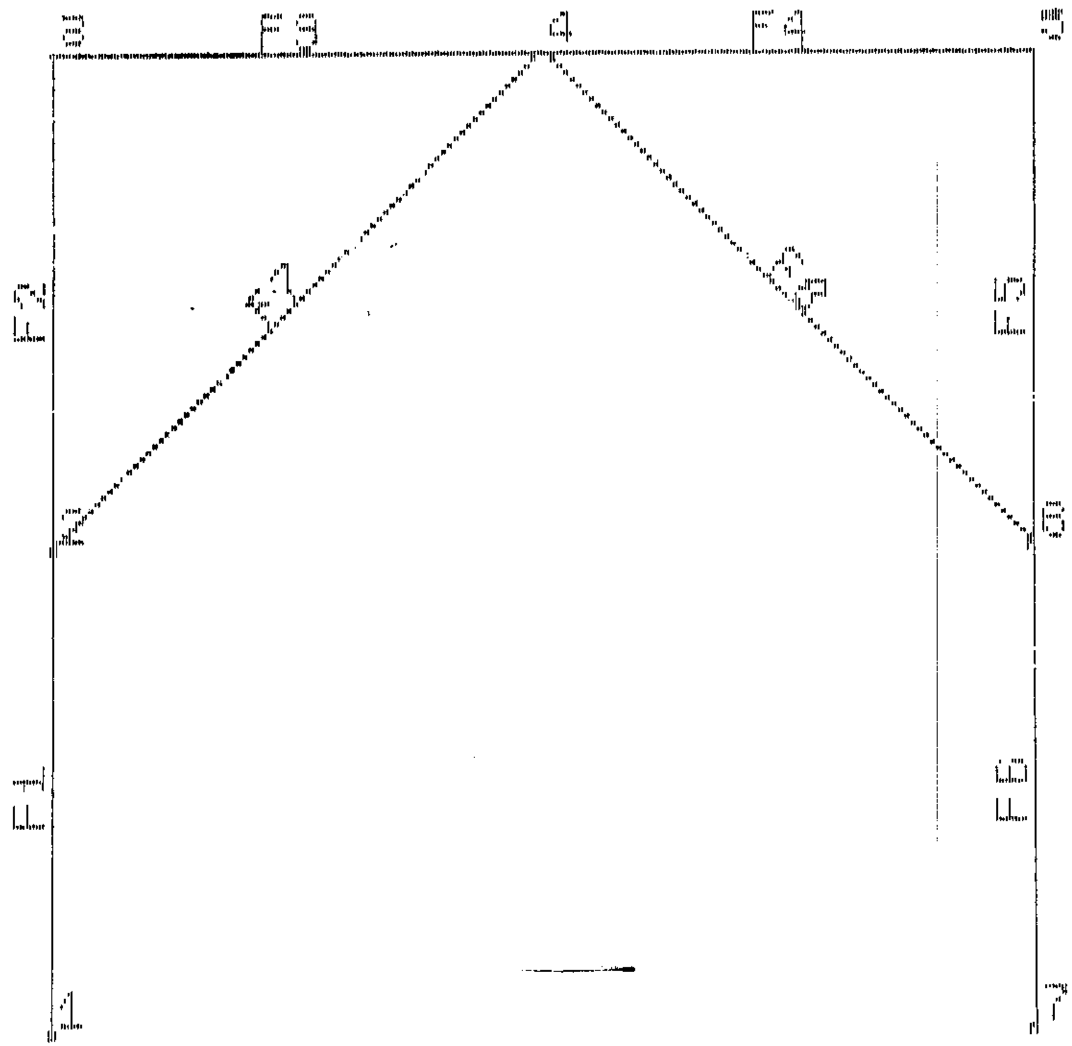
$$M_E = -5.312 + 2,466 * 10^{-3} * (-140.625) = -5.6588 \text{ mT}$$

$$\omega_E = \frac{M_E - \gamma_B M_{iD}}{\frac{4EI}{L}(1-\gamma_B \gamma_C)} - \frac{\delta}{L} = \frac{-5.6588 - \frac{1}{2}(-0.0547)}{6000\,(1-0.5*0.5)} - \frac{2.466 \times 10^{-3}}{4} =$$

$$= -1.8679 * 10^{-3} \text{ rad}$$

```
SISTEMA 1
SYSTEM    P=1
N=7    L=1
JOINTS
1      X=0.          Y=0.          Z=0.
2      X=0.          Y=4.          Z=0.
3      X=0.          Y=8.          Z=0.
4      X=4.          Y=8.          Z=0.
5      X=8.          Y=8.          Z=0.
6      X=8.          Y=4.          Z=0.
7      X=8.          Y=0.          Z=0.
:
RESTRAINS
1.7.1          R=0,0.1,1.1.0
1              R=1.1,1.1.1.1
7              R=1,1,1.1.1.1
:
FRAME
NM=1
1     A=1.E15   I=3.    E=2.1E7
1     1    2    M=1    LP=1.0
2     2    3    M=1    LP=1.0
3     3    4    M=1    LP=1.0
4     4    5    M=1    LP=1.0
5     5    6    M=1    LP=1,0
6     6    7    M=1    LP=1.0
7     2    4    M=1    LP=1.0   LP=1.1
8     4    6    M=1    LP=1.0   LR=1.1
:
LOADS
3     L=1   F=,,,,,10.
5     L=1   F=,,,,,-10.
FORCES
1      C=1.
:
```

Para evitar sing. de ka mult. por 10⁴ ⇒ Resultados por 10⁻⁴



FILE = PROB1

UNDEFORMED GEOMETRY

SAP80

* *

* * * *   J O I N T   D I S P L A C E M E N T S   * * * *

* *

LOAD CONDITION 1 - DISPLACEMENTS "U" AND ROTATIONS "R"

JOINT	U(X)	U(Y)	R(Z)
1	.0000E+00	.0000E+00	.0000E+00
2	.2480E-06	.1060E-22	-.4625E-07
3	.2760E-08	-.2603E-21	.1829E-06
4	.2760E-08	.2452E-06	-.7961E-10
5	.2760E-08	-.2662E-21	-.1826E-06
6	-.2425E-06	.3991E-23	.4513E-07
7	.0000E+00	.0000E+00	.0000E+00

R E A C T I O N S   A N D   A P P L I E D   F O R C E S

LOAD CONDITION 1 - FORCES "F" AND MOMENTS "M"

JOINT	F(X)	F(Y)	M(Z)
1	-1.8370	-.0557	4.4023
2	-.0319	-.0319	.0000
3	.0502	.0000	10.0000
4	.0670	.1205	.0000
5	.0017	.0000	-10.0000
6	-.0670	.0045	.0000
7	1.7981	-.0210	-4.3070
TOTAL	-.1883E-01	.1652E-01	.9530E-01

* *

* * F R A M E   M E M B E R   F O R C E S * * * * *

* *

.OAD COMBINATION MULTIPLIERS

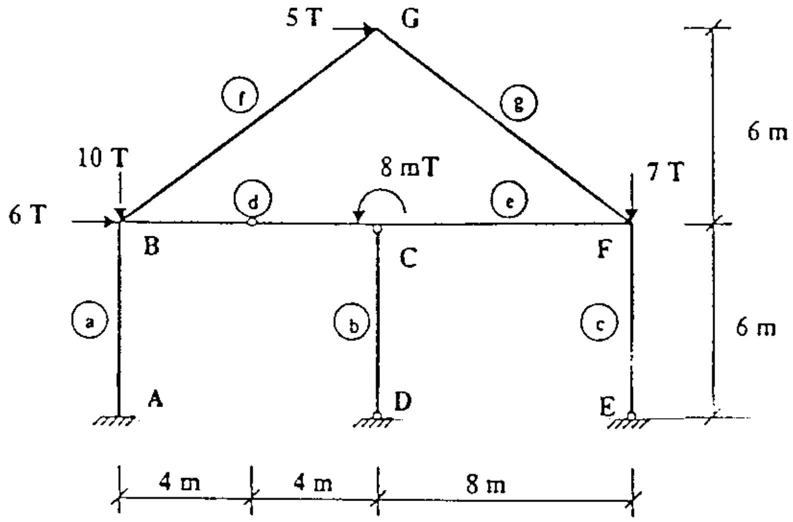
JEW LOAD   OLD LOAD CONDITION
COMB.   1
1   1.000

1EMBERS WITH NUMBERS BETWEEN   1 & 32000

MEM #	LOAD #	AXIAL FORCE	DIST I	1-2 PLANE SHEAR	1-2 PLANE MOMENT	1-3 PLANE SHEAR	1-3 PLANE MOMENT	AXIAL TORQUE
1								
	1	.06						
			.0	1.84	-4.40			
			4.0	1.84	2.95			
2								
	1	-1.42						
			.0	.33	2.95			
			4.0	.33	4.27			
3								
	1	.00						
			.0	1.42	-5.73			
			4.0	1.42	-.04			
4								
	1	.00						
			.0	-1.42	-.04			
			4.0	-1.42	-5.71			
5								
	1	-1.42						
			.0	-.35	4.29			
			4.0	-.35	2.89			
6								
	1	.02						
			.0	-1.80	2.89			
			4.0	-1.80	-4.31			
7								
	1	50126569.56						
			.0	.00	.00			
			5.7	.00	.00			
8								
	1	24480417.69						
			.0	.00	.00			
			5.7	.00	.00			

PROBLEMA 5

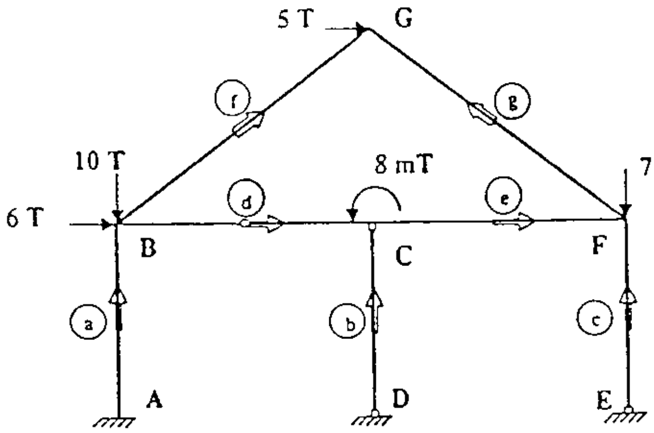
Plantear la ecuación P = K * d, para la obtención de los movimientos en todos los nudos, para la estructura reticulada plana que se indica en la figura.



Datos para todas las barras:

Sección constante de 0,30 m * 0,50 m
$E = 2{,}0 * 10^6$ T/m^2

En primer lugar se procede a realizar la numeración de nudos y barras, así como la definición de los ejes locales de cada barra de acuerdo con la figura siguiente:

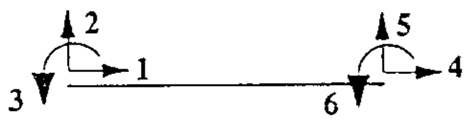


Teniendo en cuenta que:

$$EI = 2 * 10^6 T/m^2 * \frac{1}{12} 0{,}3 * 0{,}5^3 m^4 = 6{,}25 * 10^3 m^2 T$$

$$EA = 2 * 10^6 T/m^2 * 0{,}3 * 0{,}5 m^2 = 300 * 10^3 T$$

se puede pasar a formar las matrices de rigidez elementales en coordenadas locales, tomando los grados de libertad que se indican:



a) Barra de pórtico plano:

$$[K'] = \begin{bmatrix} [K'_{11}] & [K'_{12}] \\ [K'_{21}] & [K'_{22}] \end{bmatrix} = \left[\begin{array}{ccc|ccc} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ \hline -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{array}\right]$$

b) Barra de pórtico plano con una articulación en el extremo i:

Para obtenerla, se puede hacer un planteamiento de tipo general (dar movimientos unidad manteniendo nulos los demás), obteniendo directamente la matriz, o bien, eliminar M_3 mediante condensación de la matriz genérica:

$$\begin{Bmatrix} N_1 \\ V_2 \\ \hline M_3 \\ \hline N_4 \\ V_5 \\ M_6 \end{Bmatrix} = \left[\begin{array}{cc|c|ccc} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \hline 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ \hline -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{array}\right] \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \hline \vartheta_3 \\ \hline \delta_4 \\ \delta_5 \\ \vartheta_6 \end{Bmatrix}$$

Reordenando:

$$\begin{Bmatrix} N_1 \\ V_2 \\ N_4 \\ V_5 \\ M_6 \\ \hline M_3 \end{Bmatrix} = \left[\begin{array}{ccccc|c} \frac{EA}{L} & 0 & -\frac{EA}{L} & 0 & 0 & 0 \\ 0 & \frac{12EI}{L^3} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{6EI}{L^2} \\ -\frac{EA}{L} & 0 & \frac{EA}{L} & 0 & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} & \frac{2EI}{L} \\ \hline 0 & \frac{6EI}{L^2} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} & \frac{4EI}{L} \end{array}\right] \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_4 \\ \delta_5 \\ \vartheta_6 \\ \hline \vartheta_3 \end{Bmatrix}$$

que se puede escribir:

$$\begin{Bmatrix} \{P_F\} \\ \hline M_3 \end{Bmatrix} = \left[\begin{array}{c|c} [K_{FF}] & [K_{FL}] \\ \hline [K_{LF}] & K_{LL} \end{array}\right] \begin{Bmatrix} \{\delta_F\} \\ \hline \vartheta_3 \end{Bmatrix}$$

pudiéndose obtener:

$$\{P_F\} = \left([K_{FF}] - [K_{FL}]K_{LL}^{-1}[K_{LF}]\right)\{\delta_F\}$$

Operando:

$$[K_{FL}]K_{LL}^{-1}[K_{LF}]=\begin{Bmatrix} 0 \\ \frac{6EI}{L^2} \\ 0 \\ -\frac{6EI}{L^2} \\ \frac{2EI}{L} \end{Bmatrix}\frac{L}{4EI}\begin{Bmatrix} 0 & \frac{6EI}{L^2} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \end{Bmatrix}=$$

$$=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{9EI}{L^3} & 0 & -\frac{9EI}{L^3} & \frac{3EI}{L^2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{9EI}{L^3} & 0 & \frac{9EI}{L^3} & -\frac{3EI}{L^2} \\ 0 & \frac{3EI}{L^2} & 0 & -\frac{3EI}{L^2} & \frac{3EI}{L} \end{bmatrix}$$

$$[K_{FF}]-[K_{FL}]K_{LL}^{-1}[K_{LF}]=\begin{bmatrix} \frac{EA}{L} & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{3EI}{L^3} & 0 & -\frac{3EI}{L^3} & \frac{3EI}{L^2} \\ -\frac{EA}{L} & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{3EI}{L^3} & 0 & \frac{3EI}{L^3} & -\frac{3EI}{L^2} \\ 0 & \frac{3EI}{L^2} & 0 & -\frac{3EI}{L^2} & \frac{3EI}{L} \end{bmatrix}$$

y orlando de ceros, se obtiene:

$$\begin{Bmatrix} N_1 \\ V_2 \\ M_3 \\ N_4 \\ V_5 \\ M_6 \end{Bmatrix}=\begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{3EI}{L^3} & 0 & 0 & -\frac{3EI}{L^3} & \frac{3EI}{L^2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{3EI}{L^3} & 0 & 0 & \frac{3EI}{L^3} & -\frac{3EI}{L^2} \\ 0 & \frac{3EI}{L^2} & 0 & 0 & -\frac{3EI}{L^2} & \frac{3EI}{L} \end{bmatrix}\begin{Bmatrix} \delta_1 \\ \delta_2 \\ \vartheta_3 \\ \delta_4 \\ \delta_5 \\ \vartheta_6 \end{Bmatrix}$$

c) Barra de pórtico plano con articulación en ambos extremos:

Al igual que en el caso anterior se puede obtener eliminando las incógnitas correspondientes a los dos giros. Partiendo de la expresión general de una barra de pórtico plano, y reordenando filas y columnas:

$$\begin{Bmatrix} N_1 \\ V_2 \\ N_4 \\ V_5 \\ \hline M_3 \\ M_6 \end{Bmatrix} = \left[\begin{array}{cccc:cc} \frac{EA}{L} & 0 & -\frac{EA}{L} & 0 & 0 & 0 \\ 0 & \frac{12EI}{L^3} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{6EI}{L^2} \\ -\frac{EA}{L} & 0 & \frac{EA}{L} & 0 & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} & -\frac{6EI}{L^2} \\ \hdashline 0 & \frac{6EI}{L^2} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} & \frac{2EI}{L} \\ 0 & \frac{6EI}{L^2} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} & \frac{4EI}{L} \end{array}\right] \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_4 \\ \delta_5 \\ \hline \vartheta_3 \\ \vartheta_6 \end{Bmatrix}$$

$$[K_{FL}][K_{LL}]^{-1}[K_{LF}] = \begin{bmatrix} 0 & 0 \\ \frac{6EI}{L} & \frac{6EI}{L} \\ 0 & 0 \\ -\frac{6EI}{L} & -\frac{6EI}{L} \end{bmatrix} \begin{bmatrix} \frac{L}{3EI} & -\frac{L}{6EI} \\ -\frac{L}{6EI} & \frac{L}{3EI} \end{bmatrix} \begin{bmatrix} 0 & \frac{6EI}{L^2} & 0 & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & 0 & -\frac{6EI}{L^2} \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{12EI}{L^3} & 0 & -\frac{12EI}{L^3} \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & 0 & \frac{12EI}{L^3} \end{bmatrix}$$

$$[K_{FF}] - [K_{FL}][K_{LL}]^{-1}[K_{LF}] = \begin{bmatrix} \frac{EA}{L} & 0 & -\frac{EA}{L} & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{EA}{L} & 0 & \frac{EA}{L} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

y orlando de ceros se llega a:

$$\begin{Bmatrix} N_1 \\ V_2 \\ M_3 \\ \hline N_4 \\ V_5 \\ M_6 \end{Bmatrix} = \left[\begin{array}{ccc:ccc} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hdashline -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \vartheta_3 \\ \hline \delta_4 \\ \delta_5 \\ \vartheta_6 \end{Bmatrix}$$

d) Barra de pórtico plano con articulación en el centro del vano:

$$\begin{Bmatrix} N_1 \\ V_2 \\ M_3 \\ \hdashline N_4 \\ V_5 \\ M_6 \end{Bmatrix} = \left[\begin{array}{ccc:ccc} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{3EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{3EI}{L} \\ \hdashline -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{3EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{3EI}{L} \end{array}\right] \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \vartheta_3 \\ \hdashline \delta_4 \\ \delta_5 \\ \vartheta_6 \end{Bmatrix}$$

A continuación se puede montar la matriz de rigidez de la estructura, montando sólo la parte correspondiente a los desplazamientos existentes. Por lo que de acuerdo con las referencias indicadas en la primera figura resulta:

$$\begin{Bmatrix} \{P_B\} \\ \{P_C\} \\ \{P_F\} \\ \{P_G\} \end{Bmatrix} = \begin{bmatrix} [K^a_{22}]+[K^d_{11}]+[K^f_{11}] & [K^d_{12}] & 0 & [K^f_{12}] \\ [K^d_{21}] & [K^b_{22}]+[K^d_{22}]+[K^c_{11}] & [K^c_{12}] & 0 \\ 0 & [K^c_{21}] & [K^c_{22}]+[K^e_{22}]+[K^g_{11}] & [K^g_{12}] \\ [K^f_{21}] & 0 & [K^g_{21}] & [K^f_{22}]+[K^g_{22}] \end{bmatrix} \begin{Bmatrix} \{U_B\} \\ \{U_C\} \\ \{U_F\} \\ \{U_G\} \end{Bmatrix}$$

Por lo tanto lo único que resta por hacer es obtener las matrices de rigidez elementales en coordenadas globales para poder montarlas en la matriz de rigidez global indicada anteriormente, teniendo en cuenta que la matriz de cambio de coordenadas locales a globales es del tipo:

$$[L_D] = \begin{bmatrix} l_1 & l_2 & 0 \\ m_1 & m_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

resulta:

Barra a:

$$[L^a_D] = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[K^a_{22}] = [L^a_D][K^a_{22}]'[L^a_D]^T = 10^3 \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 50 & 0 & 0 \\ 0 & 0{,}347 & -1{,}042 \\ 0 & -1{,}042 & 4{,}167 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[K^a_{22}] = 10^3 \begin{bmatrix} 0{,}347 & 0 & 1{,}042 \\ 0 & 50 & 0 \\ 1{,}042 & 0 & 4{,}167 \end{bmatrix}$$

Barra b:

$$\left[L_D^b\right]=\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\left[K_{22}^b\right]=\left[L_D^b\right]\left[K_{22}^b\right]'\left[L_D^b\right]^T=10^3\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}50 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}0 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\left[K_{22}^b\right]=10^3\begin{bmatrix}0 & 0 & 0\\ 0 & 50 & 0\\ 0 & 0 & 0\end{bmatrix}$$

Barra c:

$$\left[L_D^c\right]=\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\left[K_{22}^c\right]=\left[L_D^c\right]\left[K_{22}^c\right]'\left[L_D^c\right]^T=10^3\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}50 & 0 & 0\\ 0 & 0{,}087 & -0{,}521\\ 0 & -0{,}521 & 3{,}125\end{bmatrix}\begin{bmatrix}0 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\left[K_{22}^c\right]=10^3\begin{bmatrix}0{,}087 & 0 & 0{,}521\\ 0 & 50 & 0\\ 0{,}521 & 0 & 3{,}125\end{bmatrix}$$

Barra d:

$$\left[L_D^d\right]=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\Rightarrow\left[K_{ij}^d\right]'=\left[K_{ij}^d\right]$$

$$\left[K_{11}^d\right]=10^3\begin{bmatrix}37{,}5 & 0 & 0\\ 0 & 0{,}146 & 0{,}586\\ 0 & 0{,}586 & 2{,}344\end{bmatrix}\qquad\left[K_{22}^d\right]=10^3\begin{bmatrix}37{,}5 & 0 & 0\\ 0 & 0{,}146 & -0{,}586\\ 0 & -0{,}586 & 2{,}344\end{bmatrix}$$

$$\left[K_{21}^d\right]=\left[K_{12}^d\right]^T=10^3\begin{bmatrix}-37{,}5 & 0 & 0\\ 0 & -0{,}146 & -0{,}586\\ 0 & 0{,}586 & 2{,}344\end{bmatrix}$$

Barra e:

$$\left[L_D^e\right]=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\Rightarrow\left[K_{ij}^e\right]'=\left[K_{ij}^e\right]$$

$$\left[K^e_{11}\right]=10^3\begin{bmatrix}37,5 & 0 & 0\\ 0 & 0,146 & 0,586\\ 0 & 0,586 & 3,125\end{bmatrix}\qquad \left[K^e_{22}\right]=10^3\begin{bmatrix}37,5 & 0 & 0\\ 0 & 0,146 & -0,586\\ 0 & -0,586 & 3,125\end{bmatrix}$$

$$\left[K^e_{21}\right]=\left[K^e_{12}\right]^T=10^3\begin{bmatrix}-37,5 & 0 & 0\\ 0 & -0,146 & -0,586\\ 0 & 0,586 & 1,563\end{bmatrix}$$

Barra f:

$$\left[L^f_D\right]=\begin{bmatrix}0,8 & -0,6 & 0\\ 0,6 & 0,8 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\left[K^f_{11}\right]=\left[L^f_D\right]\left[K^f_{11}\right]'\left[L^f_D\right]^T=10^3\begin{bmatrix}0,8 & -0,6 & 0\\ 0,6 & 0,8 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}30 & 0 & 0\\ 0 & 0,075 & 0,375\\ 0 & 0,375 & 2,5\end{bmatrix}\begin{bmatrix}0,8 & 0,6 & 0\\ -0,6 & 0,8 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\left[K^f_{11}\right]=10^3\begin{bmatrix}19,227 & 14,364 & -0,225\\ 14,364 & 10,848 & 0,3\\ -0,225 & 0,3 & 2,5\end{bmatrix}$$

$$\left[K^f_{22}\right]=\left[L^f_D\right]\left[K^f_{22}\right]'\left[L^f_D\right]^T=10^3\begin{bmatrix}0,8 & -0,6 & 0\\ 0,6 & 0,8 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}30 & 0 & 0\\ 0 & 0,075 & -0,375\\ 0 & -0,375 & 2,5\end{bmatrix}\begin{bmatrix}0,8 & 0,6 & 0\\ -0,6 & 0,8 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\left[K^f_{22}\right]=10^3\begin{bmatrix}19,227 & 14,364 & 0,225\\ 14,364 & 10,848 & -0,3\\ 0,225 & -0,3 & 2,5\end{bmatrix}$$

$$\left[K^f_{12}\right]=\left[K^f_{21}\right]^T=\left[L^f_D\right]\left[K^f_{12}\right]'\left[L^f_D\right]^T=10^3\begin{bmatrix}0,8 & -0,6 & 0\\ 0,6 & 0,8 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}-30 & 0 & 0\\ 0 & -0,075 & -0,375\\ 0 & -0,375 & 1,25\end{bmatrix}\begin{bmatrix}0,8 & 0,6 & 0\\ -0,6 & 0,8 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\left[K^f_{11}\right]=10^3\begin{bmatrix}19,227 & -14,364 & -0,225\\ -14,364 & -10,848 & 0,3\\ 0,225 & -0,3 & 1,25\end{bmatrix}$$

Barra g:

$$\left[L^g_D\right]=\begin{bmatrix}-0,8 & -0,6 & 0\\ 0,6 & -0,8 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\left[K_{11}^{g}\right]=\left[L_{D}^{g}\right]\left[K_{11}^{g}\right]'\left[L_{D}^{g}\right]^{T}=10^{3}\begin{bmatrix}-0,8 & -0,6 & 0\\ 0,6 & -0,8 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}30 & 0 & 0\\ 0 & 0,075 & 0,375\\ 0 & 0,375 & 2,5\end{bmatrix}\begin{bmatrix}-0,8 & 0,6 & 0\\ -0,6 & -0,8 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\left[K_{11}^{g}\right]=10^{3}\begin{bmatrix}19,227 & -14,364 & -0,225\\ -14,364 & 10,848 & -0,3\\ -0,225 & -0,3 & 2,5\end{bmatrix}$$

$$\left[K_{22}^{g}\right]=\left[L_{D}^{g}\right]\left[K_{22}^{g}\right]'\left[L_{D}^{g}\right]^{T}=10^{3}\begin{bmatrix}-0,8 & -0,6 & 0\\ 0,6 & -0,8 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}30 & 0 & 0\\ 0 & 0,075 & -0,375\\ 0 & -0,375 & 2,5\end{bmatrix}\begin{bmatrix}-0,8 & 0,6 & 0\\ -0,6 & -0,8 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\left[K_{22}^{f}\right]=10^{3}\begin{bmatrix}19,227 & -14,364 & 0,225\\ -14,364 & 10,848 & 0,3\\ 0,225 & 0,3 & 2,5\end{bmatrix}$$

$$\left[K_{12}^{g}\right]=\left[K_{21}^{g}\right]^{T}=\left[L_{D}^{f}\right]\left[K_{12}^{f}\right]'\left[L_{D}^{f}\right]^{T}=10^{3}\begin{bmatrix}-0,8 & -0,6 & 0\\ 0,6 & -0,8 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}-30 & 0 & 0\\ 0 & -0,075 & 0,375\\ 0 & -0,375 & 1,25\end{bmatrix}\begin{bmatrix}-0,8 & 0,6 & 0\\ -0,6 & -0,8 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\left[K_{11}^{f}\right]=10^{3}\begin{bmatrix}-19,227 & 14,364 & -0,225\\ 14,364 & -10,848 & -0,3\\ 0,225 & 0,3 & 1,25\end{bmatrix}$$

Por lo tanto:

$$\left[K_{22}^{a}\right]+\left[K_{11}^{d}\right]+\left[K_{11}^{f}\right]=10^{3}\begin{bmatrix}57,074 & 14,364 & 0,817\\ 14,364 & 60,994 & 0,886\\ 0,817 & 0,886 & 9,011\end{bmatrix}$$

$$\left[K_{22}^{b}\right]+\left[K_{22}^{d}\right]+\left[K_{11}^{e}\right]=10^{3}\begin{bmatrix}75 & 0 & 0\\ 0 & 50,292 & 0\\ 0 & 0 & 5,469\end{bmatrix}$$

$$\left[K_{22}^{g}\right]+\left[K_{22}^{f}\right]=10^{3}\begin{bmatrix}38,454 & 0 & 0,45\\ 0 & 21,696 & 0\\ 0,45 & 0 & 5\end{bmatrix}$$

$$\left[K_{22}^{c}\right]+\left[K_{22}^{e}\right]+\left[K_{11}^{g}\right]=10^{3}\begin{bmatrix}56,814 & -14,364 & 0,296\\ -14,364 & 60,994 & -0,886\\ 0,296 & -0,886 & 8,75\end{bmatrix}$$

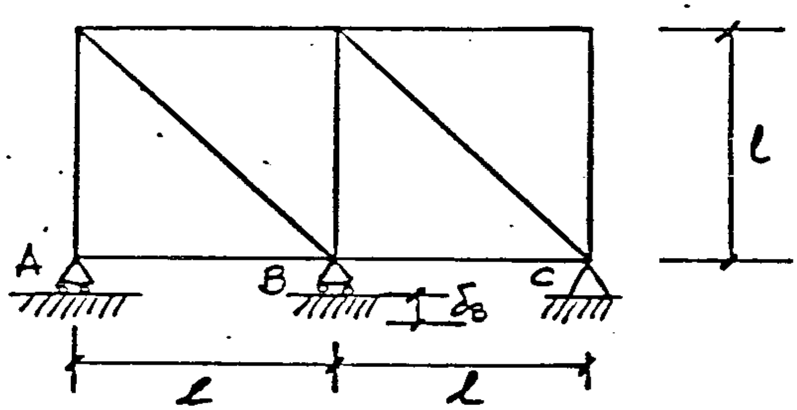
$$\begin{Bmatrix} 6 \\ -10 \\ 0 \\ 0 \\ 0 \\ 8 \\ 0 \\ -7 \\ 0 \\ 5 \\ 0 \\ 0 \end{Bmatrix} = 10^3 * \left[\begin{array}{ccc|ccc|ccc|ccc} 57{,}074 & 14{,}364 & 0{,}817 & -37{,}5 & 0 & 0 & 0 & 0 & 0 & -19{,}227 & -14{,}364 & -0{,}225 \\ 14{,}364 & 60{,}994 & 0{,}886 & 0 & -0{,}146 & 0{,}586 & 0 & 0 & 0 & -14{,}364 & -10{,}848 & 0{,}3 \\ 0{,}817 & 0{,}886 & 9{,}011 & 0 & -0{,}586 & 2{,}344 & 0 & 0 & 0 & 0{,}225 & -0{,}3 & 1{,}25 \\ \hline -37{,}5 & 0 & 0 & 75 & 0 & 0 & -37{,}5 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0{,}146 & -0{,}586 & 0 & 50{,}292 & 0 & 0 & -0{,}146 & 0{,}586 & 0 & 0 & 0 \\ 0 & 0{,}586 & 2{,}344 & 0 & 0 & 5{,}469 & 0 & -0{,}586 & 1{,}563 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & -37{,}5 & 0 & 0 & 56{,}814 & -14{,}364 & 0{,}296 & -19{,}227 & 14{,}364 & -0{,}225 \\ 0 & 0 & 0 & 0 & -0{,}146 & -0{,}586 & -14{,}364 & 60{,}994 & -0{,}886 & 14{,}364 & -10{,}848 & -0{,}3 \\ 0 & 0 & 0 & 0 & 0{,}586 & 1{,}563 & 0{,}296 & -0{,}886 & 8{,}75 & 0{,}225 & 0{,}3 & 1{,}25 \\ \hline -19{,}227 & -14{,}364 & 0{,}225 & 0 & 0 & 0 & -19{,}227 & 14{,}364 & 0{,}225 & 38{,}454 & 0 & 0{,}45 \\ -14{,}364 & -10{,}848 & -0{,}3 & 0 & 0 & 0 & 14{,}364 & -10{,}848 & 0{,}3 & 0 & 21{,}696 & 0 \\ -0{,}225 & 0{,}3 & 1{,}25 & 0 & 0 & 0 & -0{,}225 & -0{,}3 & 1{,}25 & 0{,}45 & 0 & 5 \end{array}\right] \begin{Bmatrix} u_B \\ v_B \\ \vartheta_B \\ u_C \\ v_C \\ \vartheta_C \\ u_F \\ v_F \\ \vartheta_F \\ u_G \\ v_G \\ \vartheta_G \end{Bmatrix}$$

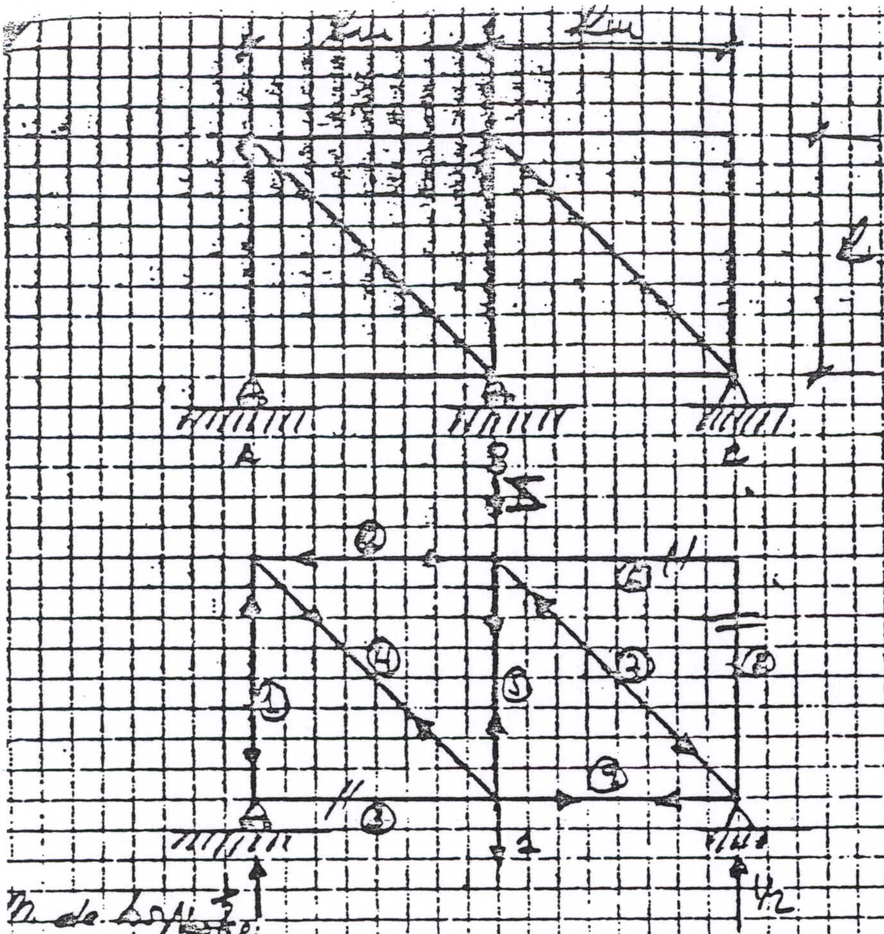
UNIVERSIDAD NACIONAL DE EDUCACION A DISTANCIA

Asignatura: ANALISIS DE ESTRUCTURAS-METODOS NUMERICOS

Problema : 2

Calcular la reacción " X " en el Punto B de la estructura representada en la figura, cuando se produce un descenso en el apoyo B de 1 cm. (δ_B = 1 cm). Todas las barras tienen el mismo módulo de elasticidad (E) y la misma sección (A).



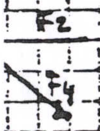


Barre	F_i	l_i	$l_i F_i^2$
1	$-1/2$	l	$l\ 1/4$
2	$-1/2$	l	$l\ 1/4$
3	0	l	0
4	$\sqrt{2}/2$	$\sqrt{2}\,l$	$l\ \sqrt{2}/2$
5	$1/2$	l	$l\ 1/4$
6	0	l	0
7	$-\sqrt{2}/2$	$\sqrt{2}\,l$	$l\ \sqrt{2}/2$
8	0	l	0
9	$1/2$	l	$l\ 1/4$

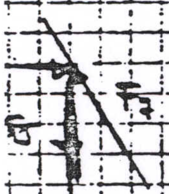
$\Sigma = l(1+\sqrt{2})$

m. de [illegible] :

$F_1 = \frac{1}{2}$; $F_3 = 0$ $\qquad F_6 = F_8 = 0$



$\Rightarrow \quad F_4 = \frac{1}{2}\,\frac{1}{\sqrt{2}/2} = \frac{\sqrt{2}}{2}$; $F_2 = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{1}{2}$



$\Rightarrow \quad F_7 = \frac{1}{2}\,\frac{1}{\sqrt{2}/2} = \frac{\sqrt{2}}{2}$; $F_5 = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{1}{2}$

$\Rightarrow \quad F_9 = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{1}{2}$

$$\Delta l_i = \frac{F_i X l_i}{A_i E}$$

$$\sum \left(F_i X \frac{l_i}{A_i E} \right) F_i = \delta_B = \Delta = \frac{X}{AE} \sum F_i^2 l_i$$

$$X = \frac{\delta_B A E}{\sum F_i^2 l_i} = \frac{\delta_B A E}{l(1+\sqrt{2})} = \frac{0{,}01 \cdot E \cdot A}{l(1+\sqrt{2})}$$

[illegible]

UNIVERSIDAD NACIONAL DE EDUCACION A DISTANCIA

Asignatura: ANALISIS DE ESTRUCTURAS-METODOS NUMERICOS

Problema 3 :

En la estructura de la figura se produce un incremento de temperatura en todas las barras de 40° C.

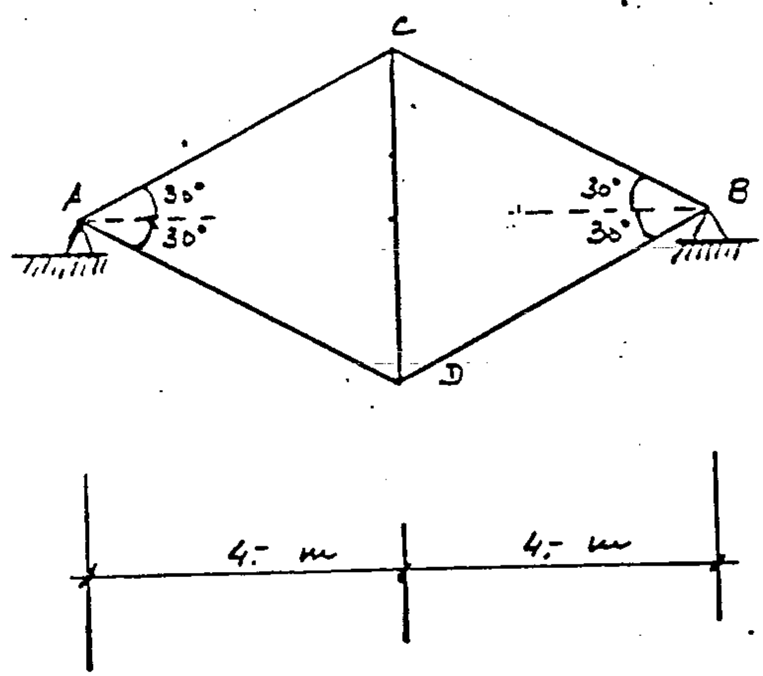
Calcular:

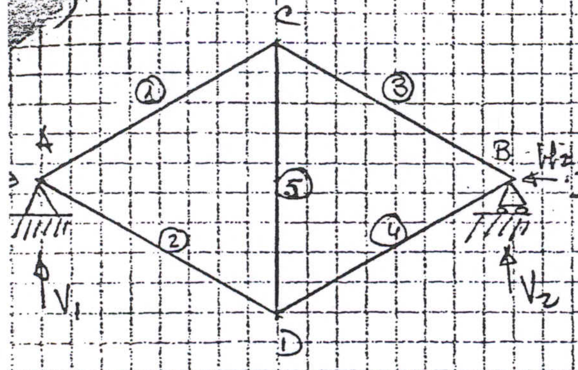
a)los esfuerzos en las barras y reacciones en los apoyos.

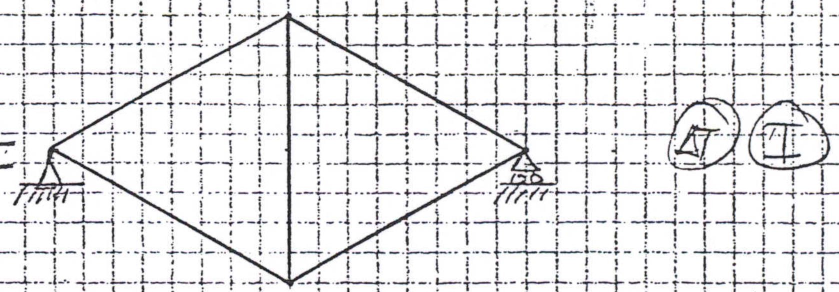
b)el desplazamiento vertical del punto C

DATOS: $\alpha = 12 \times 10^{-6}$ °C^{-1} ‖ Area igual para todas las barras(A) A= 2 cm^2

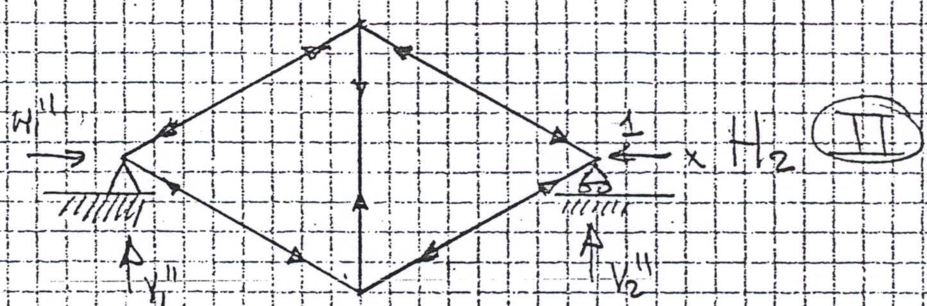
$E = 2{,}1 \times 10^{6}$ Kg/ cm^2





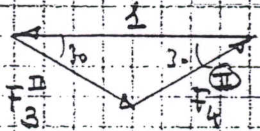


+

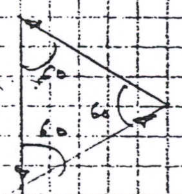


(I) ⇒ $F_i^{I} = 0$

(II) ⇒ $H_1'' = 1$, $V_1'' = V_2'' = 0$



$F_3^{II} = F_4^{II} = \frac{1}{2} \cdot \frac{1}{\cos 30} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$



$F_1^{II} = \frac{1}{\sqrt{3}}$

$F_2^{II} = \frac{1}{\sqrt{3}}$

$F_5^{II} = \frac{1}{\sqrt{3}}$

− → COMPRESIÓN
+ → TRACCIÓN

BARRA	l_i	F_i^{I}	F_i^{II}	Δl_{real_i}	$\Delta l_{real_i} \times F_i^{II}$
1	l	0	$-\sqrt{3}/3$	$-\frac{H_2 l}{\sqrt{3} AE} + \alpha \Delta T l$	$\frac{H_2 l}{3AE} - \frac{\sqrt{3}}{3} \alpha \Delta T l$
2	l	0	$-\sqrt{3}/3$	"	"
3	l	0	$-\sqrt{3}/3$	"	"
4	l	0	$-\sqrt{3}/3$	"	"
5	l	0	$\sqrt{3}/3$	$\frac{H_2 l}{\sqrt{3} AE} + \alpha \Delta T l$	$\frac{H_2 l}{3AE} + \frac{\sqrt{3}}{3} \alpha \Delta T l$

$\Delta l_{real_i} = H_2 \frac{F_i^{II} l_i}{A_i E} + \alpha \Delta T l_i$ //

(2)

Desplazamiento horizontal del punto B = 0

$$0 = \sum_{i=1}^{5} \Delta l_{real_i} \times F_i^{II} = 4\frac{H_2 l}{3AE} - 4\frac{\sqrt{3}}{3}\alpha\Delta T l + \frac{H_2 l}{3AE} + \frac{\sqrt{3}}{3}\alpha\Delta T l$$

F_i^{II}: que son las fuerzas debidas a la carga 1 colocada en B horizontalmente.

$$= 5\frac{H_2 l}{3AE} - 3\frac{\sqrt{3}}{3}\alpha\Delta T l = 0$$

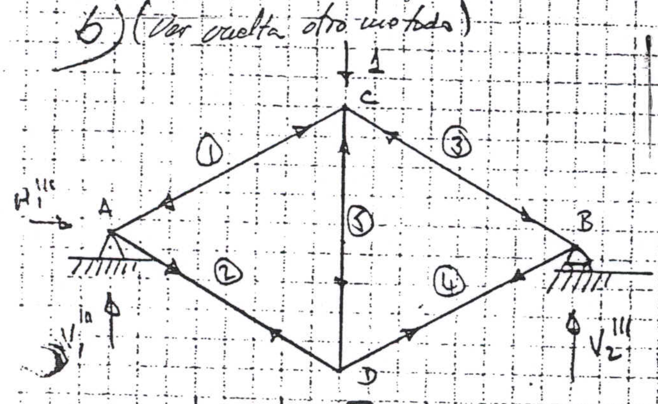
$$\frac{5 H_2 l}{3AE} = \sqrt{3}\,\alpha\Delta T l$$

$$H_2 = \frac{3\sqrt{3}}{5}\alpha\Delta T AE =$$

$$H_2 = \frac{3\sqrt{3}}{5}\, 12\times 10^{-6} \times 40 \times 2 \times 2{,}1\times 10^{6} =$$

$$= \underline{\underline{2095{,}09 \text{ Kg}}}$$

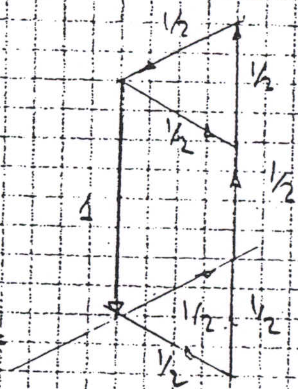
b) (Ver vuelta otro método)



$$l = \frac{4}{\cos 30} = \frac{4}{\sqrt{3}/2} = \frac{8}{\sqrt{3}}$$

$H_1^{III} = 0$

$V_1^{III} = V_2^{III} = \frac{1}{2}$



BARRA	l_i	F_i^{III}	Δl_{real_i}
1	l	$-1/2$	$-\frac{H_2 l}{\sqrt{3}AE} + \alpha\Delta T l$
2	l	$1/2$	"
3	l	$-1/2$	"
4	l	$1/2$	"
5	l	$-1/2$	$\frac{H_2 l}{\sqrt{3}AE} + \alpha\Delta T l$

$\boxed{-0{,}0017736}$

$$\delta_c = \sum F_i^{III} \times \Delta l_{real_i} = -\frac{1}{2}\left(-\frac{H_2 l}{\sqrt{3}AE} + \alpha\Delta T l\right) + \frac{1}{2}\left(-\frac{H_2 l}{\sqrt{3}AE} + \alpha\Delta T l\right) - \frac{1}{2}\left(-\frac{H_2 l}{\sqrt{3}AE} + \alpha\Delta T l\right)$$

$$+ \frac{1}{2}\left(-\frac{H_2 l}{\sqrt{3}AE} + \alpha\Delta T l\right) - \frac{1}{2}\left(\frac{H_2 l}{\sqrt{3}AE} + \alpha\Delta T l\right) =$$

$$= -\frac{1}{2}\frac{H_2 l}{\sqrt{3}AE} - \frac{1}{2}\alpha\Delta T l = -\frac{3\sqrt{3}}{5}\alpha\Delta T AE \frac{1}{2}\frac{l}{\sqrt{3}AE} - \frac{1}{2}\alpha\Delta T l = -\frac{4}{5}\alpha\Delta T l =$$

2. Al ser simétrica se calcula el alargamiento de la barra 5 y se divide por 2 obteniendo el despl. pedido.

$$\delta_5 = \frac{1}{2}\left\{\frac{T_5 l_5}{AE} + \alpha \Delta T \, l_5\right\} = \frac{1}{2}\left\{\frac{210 \times 461{,}88}{2 \times 2{,}1 \times 10^6}\right\} = \frac{0{,}355}{2} = 0{,}17736\,cm \uparrow$$

UNIVERSIDAD NACIONAL DE EDUCACION A DISTANCIA

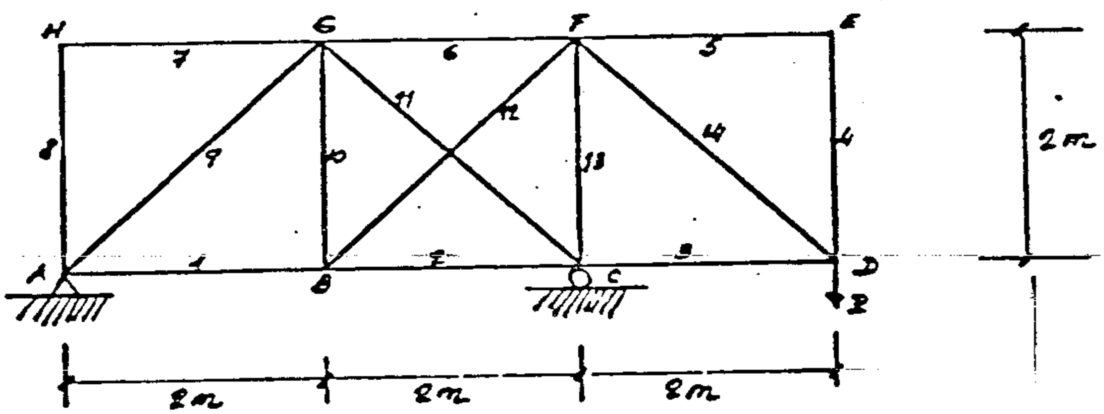
Asignatura : ANALISIS DE ESTRUCTURAS - METODOS NUMERICOS

Problema : 4

Sabiendo que la estructura de la figura se ha montado con la barra 12 un centímetro más corta; calcular la fuerza vertical P necesaria para que después del montaje el desplazamiento vertical del punto sea cero.

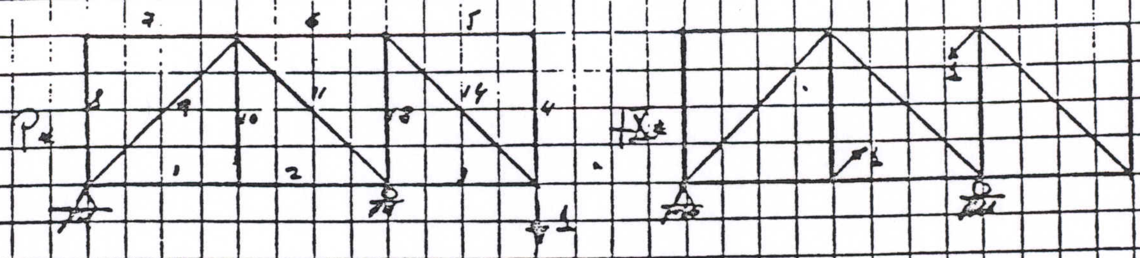
atos: $E = 2 \times 10^6$ Kg/cm^2

$A = 10$ cm^2 en todas las barras



①

PROBLEMA:



(II) (I)

$EA = 2\times10^7$

BARRAS	L_i	N_i^I	N_i^{II}	$L_i N_i^{II\,2}/100$	$L_i N_i^{II} N_i^I/100$	$L_i N_i^{I\,2}$
1	200	0	$-1/2$	$1/2$	0	0
2	200	$-\sqrt{2}/2$	$-1/2$	$1/2$	$\sqrt{2}/2$	1
3	200	0	-1	2	0	0
4	200	0	0	0	0	0
5	200	0	0	0	0	0
6	200	$-\sqrt{2}/2$	1	2	$-\sqrt{2}$	1
7	200	0	0	0	0	0
8	200	0	0	0	0	0
9	$200\sqrt{2}$	0	$\sqrt{2}/2$	$\sqrt{2}$	0	0
10	200	$-\sqrt{2}/2$	0	0	0	1
11	$200\sqrt{2}$	1	$-\sqrt{2}/2$	$\sqrt{2}$	-2	$2\sqrt{2}$
12	$200\sqrt{2}$	—	—	—	—	—
13	200	$-\sqrt{2}/2$	-1	2	$\sqrt{2}$	1
14	$200\sqrt{2}$	0	$\sqrt{2}$	$4\sqrt{2}$	0	0
				$\Sigma = 7+6\sqrt{2}$	$\Sigma = -2+\frac{\sqrt{2}}{2}$	$\Sigma = 4+2\sqrt{2}$

$\delta_0 = 0 \qquad 0 = P(7+6\sqrt{2}) + X\left(-2+\frac{\sqrt{2}}{2}\right)$

$$0 = P(7+6\sqrt{2}) + X\left(\frac{-4+\sqrt{2}}{2}\right) \Rightarrow \boxed{0 = 2(7+6\sqrt{2})P + (-4+\sqrt{2})X}$$

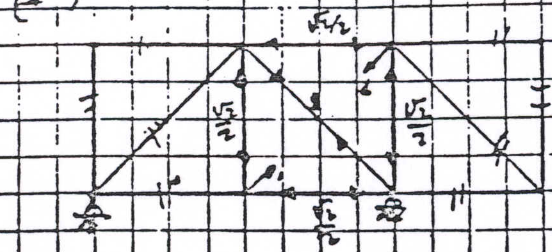
$$\delta_{12\,cálc} = \left[P\left(-2+\frac{\sqrt{2}}{2}\right) + X(4+2\sqrt{2})\right]\frac{1}{2\times10^5}$$

$$\delta_{12\,barra} = -\frac{X\,200\sqrt{2}}{2\times10^7} + \delta = -\frac{2X\sqrt{2}}{2\cdot10^5} + \delta^{=1}$$

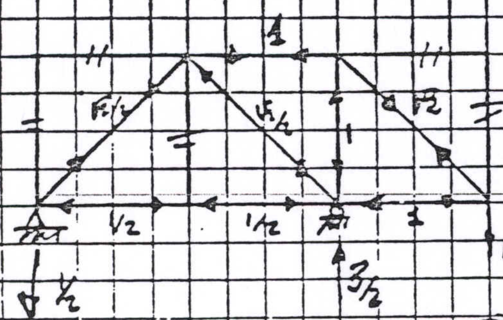
$$\left[P\,\frac{-4+\sqrt{2}}{2} + X(4+2\sqrt{2}) + 2\sqrt{2}X\right]\frac{1}{2\times10^5} = 1$$

$$\boxed{P(-4+\sqrt{2}) + 8X(1+\sqrt{2}) = 4\times10^5}$$

(I)



(II)



$$2(7+6\sqrt{2})P + (-4+\sqrt{2})X = 0$$
$$(-4+\sqrt{2})P + 8(1+\sqrt{2})X = 4\times10^5$$
$$\Rightarrow X = -2\frac{(7+6\sqrt{2})}{-4+\sqrt{2}}P$$

Sust. a la otra

$$P = \frac{4\times10^5}{\left[-4+\sqrt{2}+16\frac{(1+\sqrt{2})(7+6\sqrt{2})}{4-\sqrt{2}}\right]} = 1748,72\ \text{Kg} \Rightarrow \underline{1,7487\ T}$$

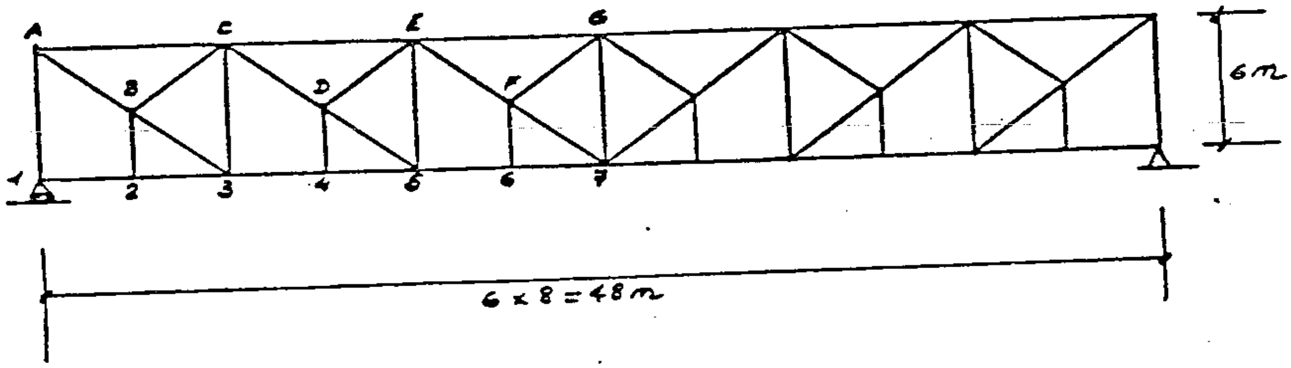
$$X = +20944.8\ \text{Kg} = \underline{+20,945\ T}$$

UNIVERSIDAD NACIONAL DE EDUCACION A DISTANCIA

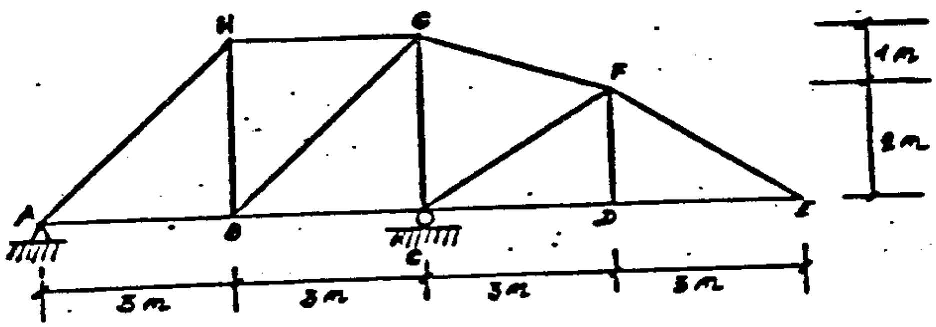
Asignatura: ANALISIS DE ESTRUCTURAS-METODOS NUMERICOS

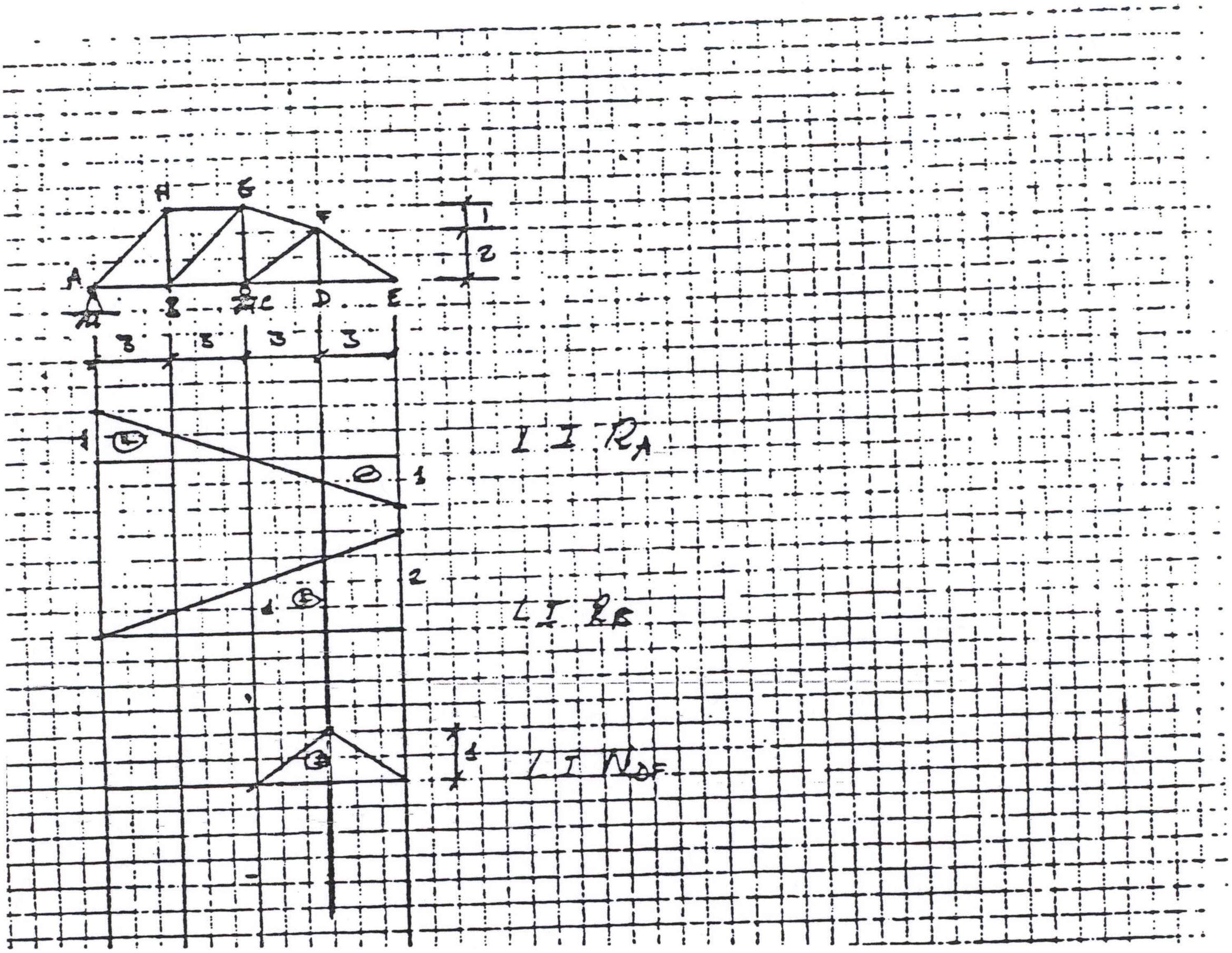
Problema 5 :

a) Calcular las líneas de influencia de los esfuerzos en las barras CE, 4D, DE, y E5 , de la estructura representada en la figura, cuando una carga unidad recorre el cordón inferior(transmisión de modo indirecto).



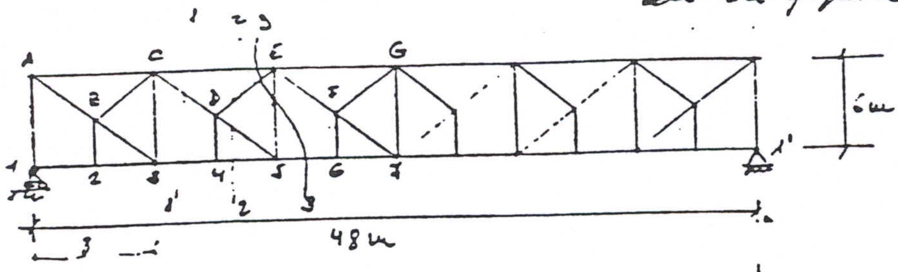
Calcùlar las LINEAS DE INFLUENCIA de las reacciones en A y C , y esfuerzo de la barra DF ,cuando una carga unidad recorre el cordón erior.

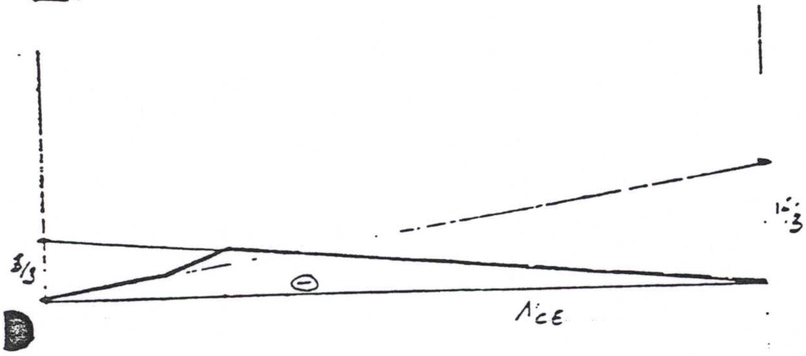



H
G
F
A
B
C
D
E
1
2
3
3
3
3
1
1
L I R_A
2
1
L I R_B
1
L I N_{DF}

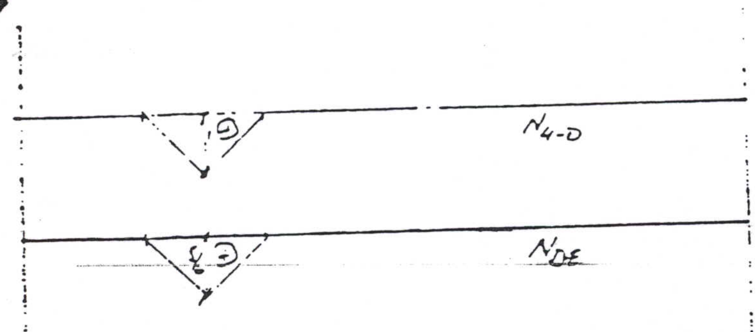
RES

PROBL 1: Calcular las líneas de influencia de los esfuerzos en las barras $\overline{CE}$, $\overline{4D}$, $\overline{DE}$ y $\overline{ES}$ de la estructura representada en la figura, cuando una carga unidad recorre el cordón inferior (Transmisión de modo indirecto).





Corte 1-1 $\sum M_E = 0$ $\begin{cases} N_{CE} \times 6 + V_1 \times 16 = 0 \; ; \; N_{CE} = -\frac{8}{3} V_1 \\ N_{CE} \times 6 + V_{11} \times 32 = 0 \; ; \; N_{CE} = -\frac{16}{3} V_{11} \end{cases}$



Inmediato

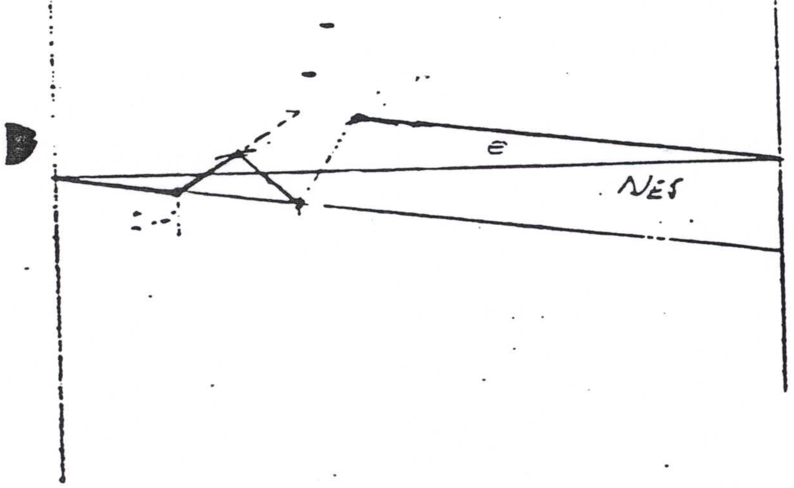
Con el eq. de nudo 4

~~o también: Corte 2-2, $\sum M_E$, para sección~~

sabiendo que con la carga en 4

$N_{CE} = -2$ y $V_1 = \frac{1}{4}$

$N_{DE}\, 6 \cos\alpha + \frac{1}{4}\, 32 - 2 \times 6 = 0 \Rightarrow N_{DE} = \frac{5}{6}$



Corte 3-3:

Parte izq. $\sum F_y^{izq} = V_1 + N_{5E} + N_{DE}\, ~~\cos\alpha~~ = 0$

$N_{5E} = -V_1 - N_{DE} \cos\alpha$

Parte dcha. $\sum F_y^{dcha} = V_{11} - N_{5E} - N_{DE} \operatorname{sen}\alpha = 0$

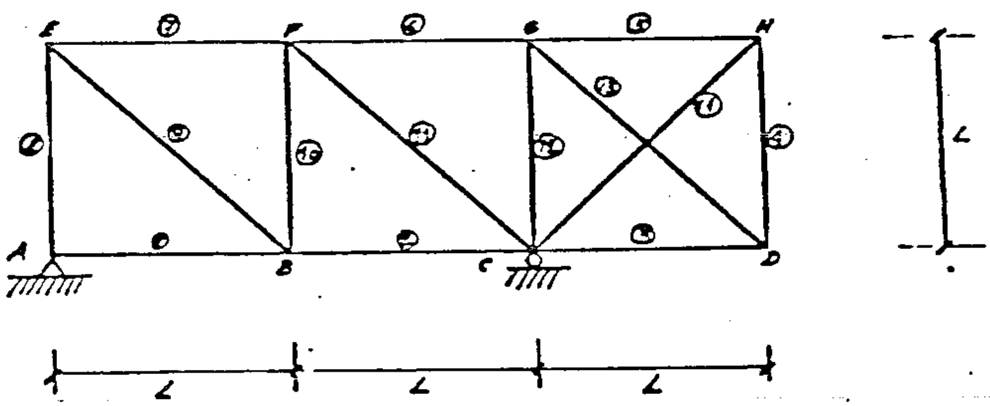
$N_5 = V_{11} - N_{DE} \operatorname{sen}\alpha$

UNIVERSIDAD NACIONAL DE EDUCACION A DISTANCIA

Asignatura : ANALISIS DE ESTRUCTURAS - METODOS NUMERICOS

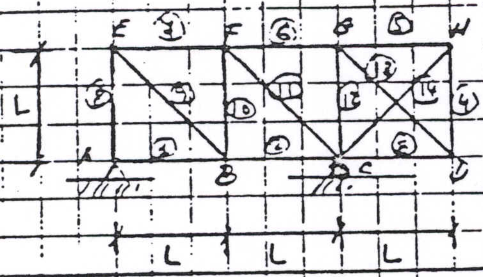
Problema : 6

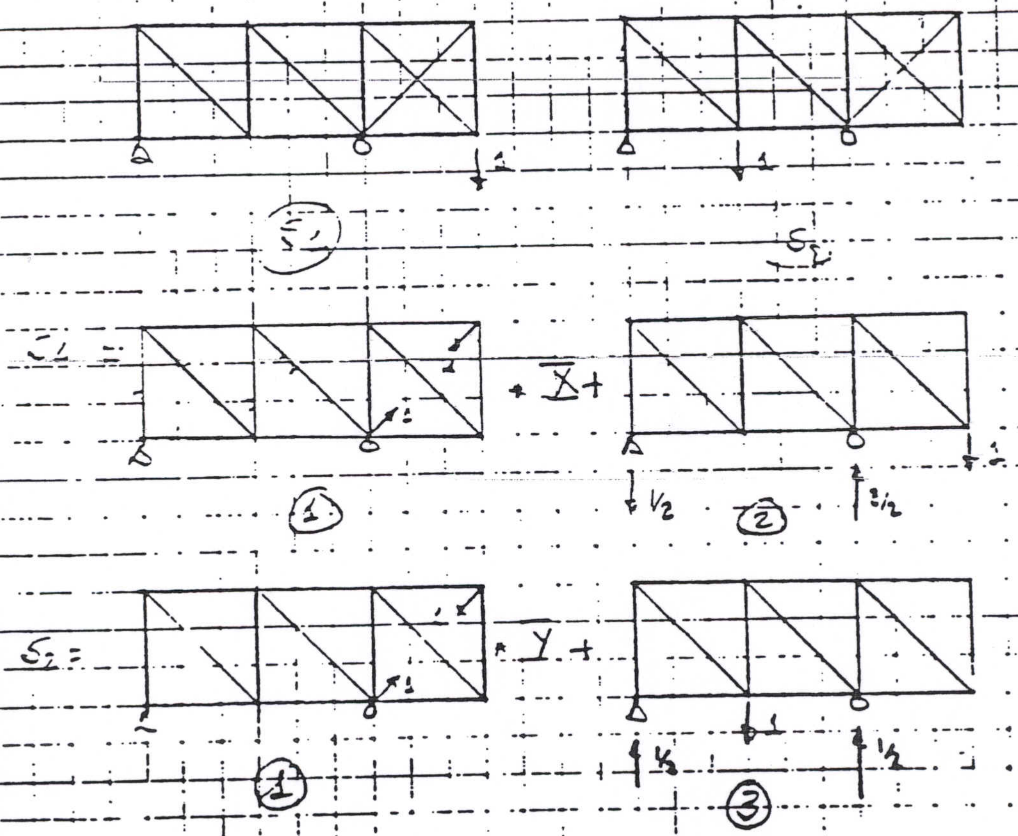
-Calcular en la estructura representada en la figura, las líneas de influencia de la reacción en A y de los esfuerzos axiles en las barras EB y GD, cuando una carga unidad recorre el cordón inferior.



Reserva Sep.

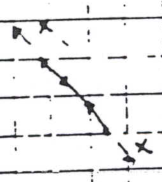
PROBLEMA: Calcular en la estructura representada en la figura, las líneas de influencia de la reacción en A y de los esfuerzos axiles en las barras EB y GD, cuando una carga unidad recorre el cordón inferior.





BARRA	l_i/L	$F_i^{(1)}$	$F_i^{(2)}$	$F_i^{(3)}$	$F_i^{(1)2}\, l_i/L$	$F_i^{(1)} F_i^{(2)}\, l_i/L$	$F_i^{(1)} F_i^{(3)}\, l_i/L$
1	1	0	0	0	0	0	0
2	1	0	$-1/2$	$1/2$	0	0	0
3	1	$-\sqrt{2}/2$	-1	0	$1/2$	$\sqrt{2}/2$	0
4	1	$-\sqrt{2}/2$	0	0	$1/2$	0	0
5	1	$-\sqrt{2}/2$	0	0	$1/2$	0	0
6	1	0	1	0	0	0	0
7	1	0	$1/2$	$-1/2$	0	0	0
8	1	0	$1/2$	$-1/2$	0	0	0
9	$\sqrt{2}$	0	$-\sqrt{2}/2$	$\sqrt{2}/2$	0	0	0
10	1	0	$1/2$	$1/2$	0	0	0
11	$\sqrt{2}$	0	$-\sqrt{2}/2$	$-\sqrt{2}/2$	0	0	0
12	1	$-\sqrt{2}/2$	-1	0	$1/2$	$\sqrt{2}/2$	0
13	$\sqrt{2}$	1	$\sqrt{2}$	0	$\sqrt{2}$	2	0
14	$\sqrt{2}$	—	—	—			
					$\Sigma = 2+\sqrt{2}$	$\Sigma = 2+\sqrt{2}$	0

(S1)



$$\left\{ \begin{aligned} u_{14}^{I} &= -X \frac{L\sqrt{2}}{AE} = -X\sqrt{2}\,\frac{L}{AE} \\ u_{14}^{II} &= \Sigma \left(X N_i^{(1)} + N_i^{(2)} \right) N_i^{(1)} \frac{l_i}{AE} \end{aligned} \right\} \quad u_{14}^{I} = u_{14}^{II}$$

$$-X\sqrt{2}\,\frac{L}{AE} = \frac{L\,\Sigma}{AE} \; X N_i^{(1)2} \frac{l_i}{L} + N_i^{(1)} N_i^{(2)} \frac{l_i}{L}$$

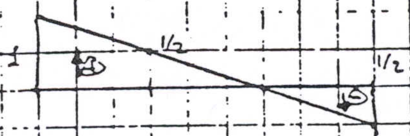
$$-X\sqrt{2} = X(2+\sqrt{2}) + 2+\sqrt{2} \Rightarrow \boxed{X = -\frac{2+\sqrt{2}}{2+2\sqrt{2}}}$$

(S2) Analogamente: $-X\sqrt{2} = X(2+\sqrt{2}) + 0 \Rightarrow \boxed{Y = 0}$

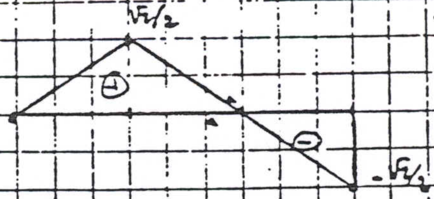
(M)

	S_1	S_2
R_A	$1/2$	$1/2$
$N_{EB} = N_3$	$X N_3^{(1)} + N_3^{(2)} = -\sqrt{2}/2$	$\sqrt{2}/2$
$N_{GD} = N_{13}$	$X N_{13}^{(1)} + N_{13}^{(2)} = -\frac{2+\sqrt{2}}{2+2\sqrt{2}} + \sqrt{2} = \frac{\sqrt{2}}{2}$	0

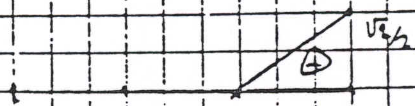
$LI \; R_A$:



$LI \; N_{EB} = N_9$:



$LI \; N_{GD} = N_{13}$:



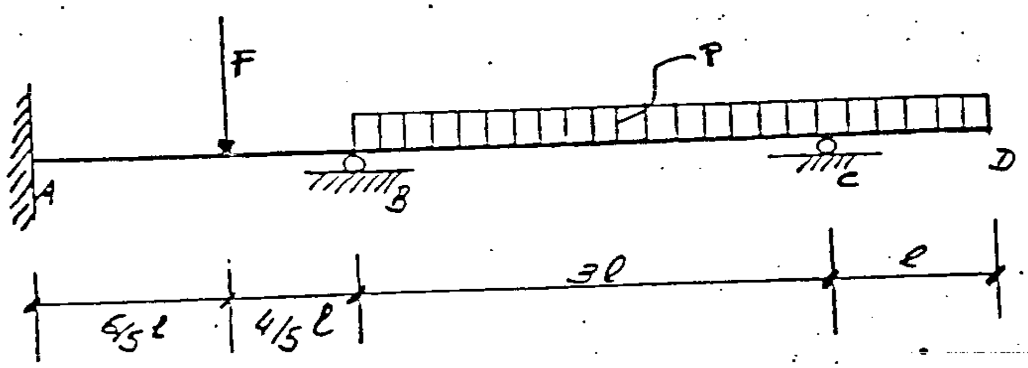
UNIVERSIDAD NACIONAL DE EDUCACION A DISTANCIA

Asignatura: ANALISIS DE ESTRUCTURAS-METODOS NUMERICOS

Problema : 1

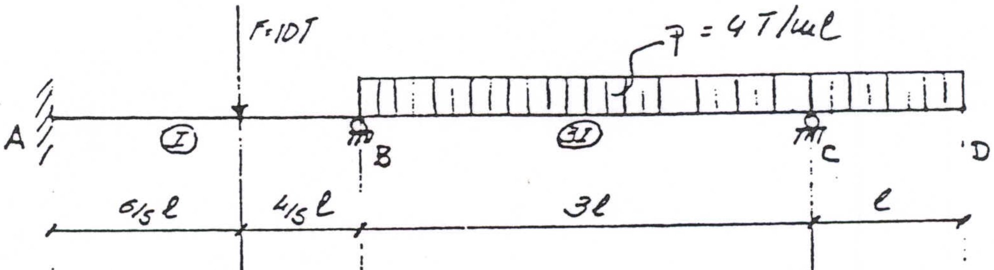
Determinar las leyes de momentos flectores, cortantes, y reacciones de la viga de la figura.

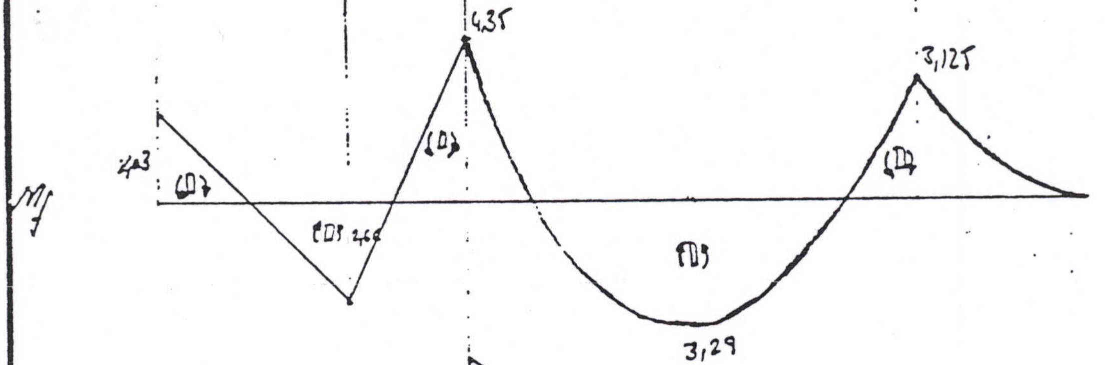
Sabiendo que : $l=1,25$ m , que la inercia del tramo AB es I , y la inercia del tramo BC es 3I . La fuerza puntual F es de 10 T, y la carga distribuida es de P=4 T/m

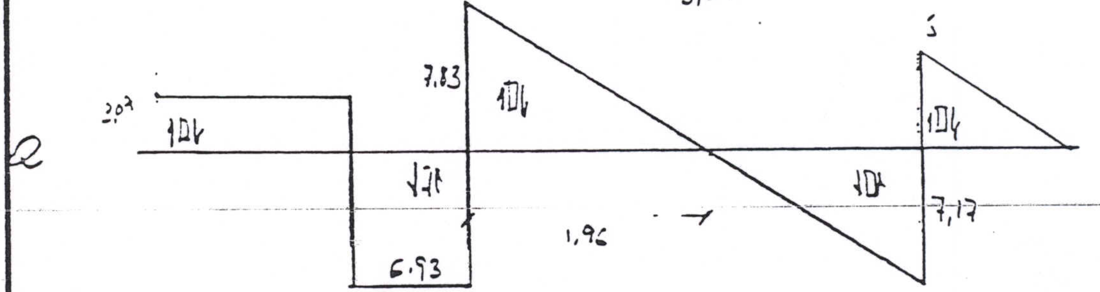


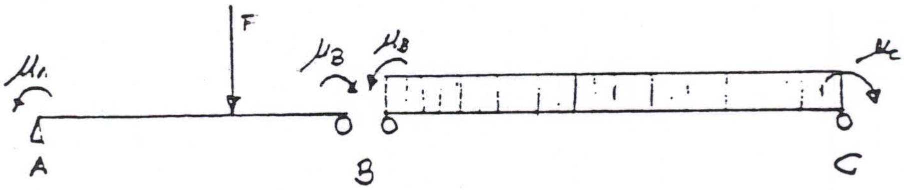
PROBLEMA 17

$l = 1,25\ m$



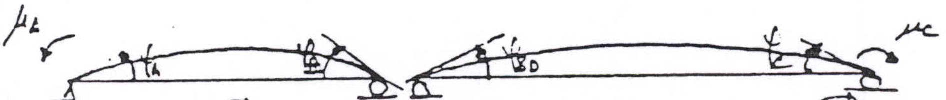






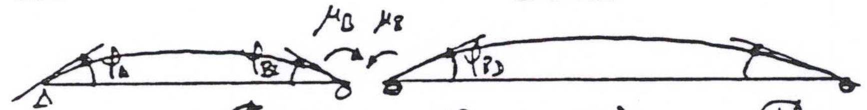
$$M_C = \frac{p l^2}{2} = \frac{4 \times 1{,}25^2}{2} = 3{,}125$$

1°



$$\varphi_A = \frac{M_A(2l)}{3EI} \qquad \varphi_{BI} = \frac{M_A(2l)}{6EI} \qquad \varphi_{BD} = \frac{M_C(3l)}{6E(3I)} \qquad \varphi_C = \frac{M_C(3l)}{3E(3I)}$$

2°



$$\varphi_A = \frac{M_B(2l)}{6EI} \qquad \varphi_{BI} = \frac{M_B(2l)}{3EI} \qquad \varphi_{BD} = \frac{M_B(3l)}{3E(3I)} \qquad \varphi_C = \frac{M_B(3l)}{6E(3I)}$$

3°

$$\varphi_A = \frac{28Fl^2}{125EI} \qquad \varphi_{BI} = \frac{32Fl^2}{125EI} \qquad \varphi_{BD} = \frac{p(3l)^3}{24E(3I)} \qquad \varphi_C = \frac{p(3l)^3}{24E(3I)}$$

PROBLEMA 1

NUDO A: Como es un empotramiento, el giro será nulo.

$$\varphi_{A_1} + \varphi_{A_2} + \varphi_{A_3} = 0$$

$$\frac{\mu_A\, 2\ell}{3EI} + \frac{\mu_B\, 2\ell}{6EI} - \frac{28 F \ell^2}{125 EI} = 0$$

$$\boxed{2\mu_A + \mu_B = \frac{84 F \ell}{125} = \frac{84 \times 10 \times 1{,}25}{125} = 8{,}4}$$

NUDO B: Por continuidad, el giro a la derecha y a la izquierda iguales.

$$\varphi_{BI} = -\varphi_{BD}$$

$$\varphi_{BI} = \varphi_{BI_1} + \varphi_{BI_2} + \varphi_{BI_3} =$$

$$= -\frac{\mu_A\, 2\ell}{6EI} - \frac{\mu_B\, 2\ell}{3EI} + \frac{32 F\ell^2}{125 EI} = \frac{\ell}{EI}\left[\frac{32F\ell}{125} - \frac{2\mu_B + \mu_A}{3}\right]$$

$$\varphi_{BD} = \varphi_{BD_1} + \varphi_{BD_2} + \varphi_{BD_3} =$$

$$= -\frac{\mu_C\, 3\ell}{6E\,3I} - \frac{\mu_B\, 3\ell}{3E\,3I} + \frac{P 27 \ell^3}{24 E\,3I} = \frac{\ell}{EI}\left[\frac{3P\ell^2}{8} - \frac{2\mu_B + \mu_C}{6}\right]$$

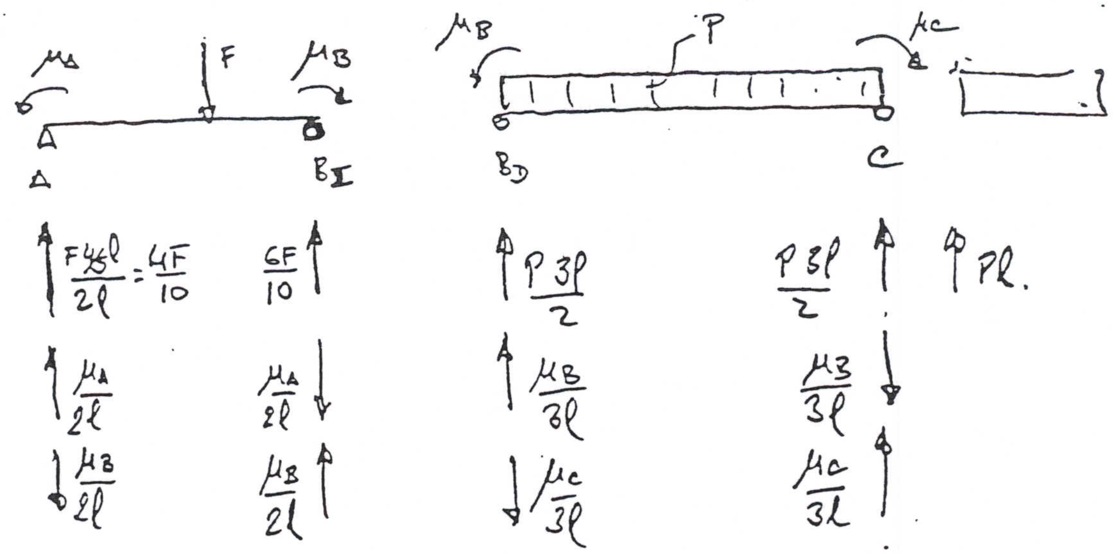
$$\frac{32 F\ell}{125} - \frac{2\mu_B + \mu_A}{3} = -\frac{3P\ell^2}{8} + \frac{2\mu_B + \mu_C}{6}$$

$$\boxed{3\mu_B + \mu_A = \frac{96 F\ell}{125} + \frac{7P\ell^2}{8}} = \frac{96 \times 10 \times 1{,}25}{125} + \frac{7 \times 4 \times 1{,}25^2}{8} = 15{,}07$$

$$\left.\begin{array}{l} 2\mu_A + \mu_B = 8{,}4 \\ \mu_A + 3\mu_B = 15{,}07 \end{array}\right\} \Rightarrow$$

$\mu_B = 4{,}35$ mT
$\mu_A = 2{,}03$ mT

PROBLEMA 1



$$\uparrow R_A = \frac{4 \times F}{10} + \frac{\mu_A}{2l} - \frac{\mu_B}{2l} = 4 + \frac{2{,}03}{2 \times 1{,}25} - \frac{4{,}35}{2 \times 1{,}25} = \underline{\underline{3{,}07\ T}}$$

$$\uparrow R_B = \left[6 + \frac{4{,}35}{2 \times 1{,}25} - \frac{2{,}03}{2 \times 1{,}25} \right] + \left\{ \frac{4 \times 3 \times 1{,}25}{2} + \frac{4{,}35}{3 \times 1{,}25} - \frac{3{,}125}{3 \times 1{,}25} \right\} =$$

$$= 6{,}93 + 7{,}83 = \underline{\underline{14{,}76\ T}}$$

$$\uparrow R_C = \left[\frac{4 \times 3 \times 1{,}25}{2} + \frac{3{,}125}{3 \times 1{,}25} - \frac{4{,}35}{3 \times 1{,}25} \right] + 4 \times 1{,}25 =$$

$$= 7{,}17 + 5 = \underline{\underline{12{,}17\ T}}$$

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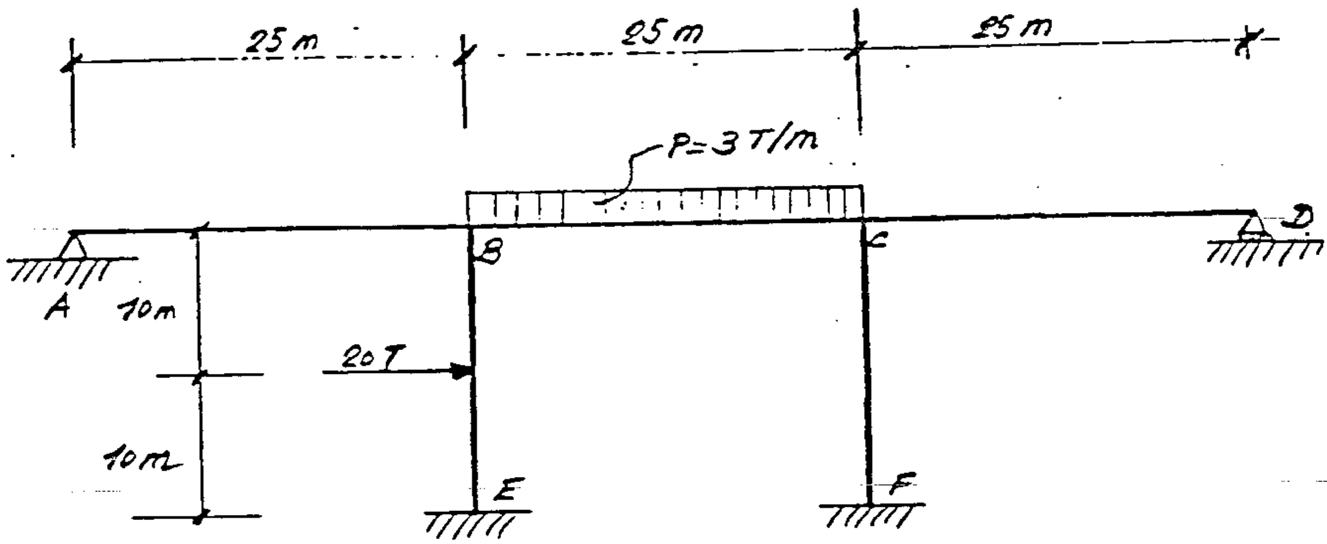
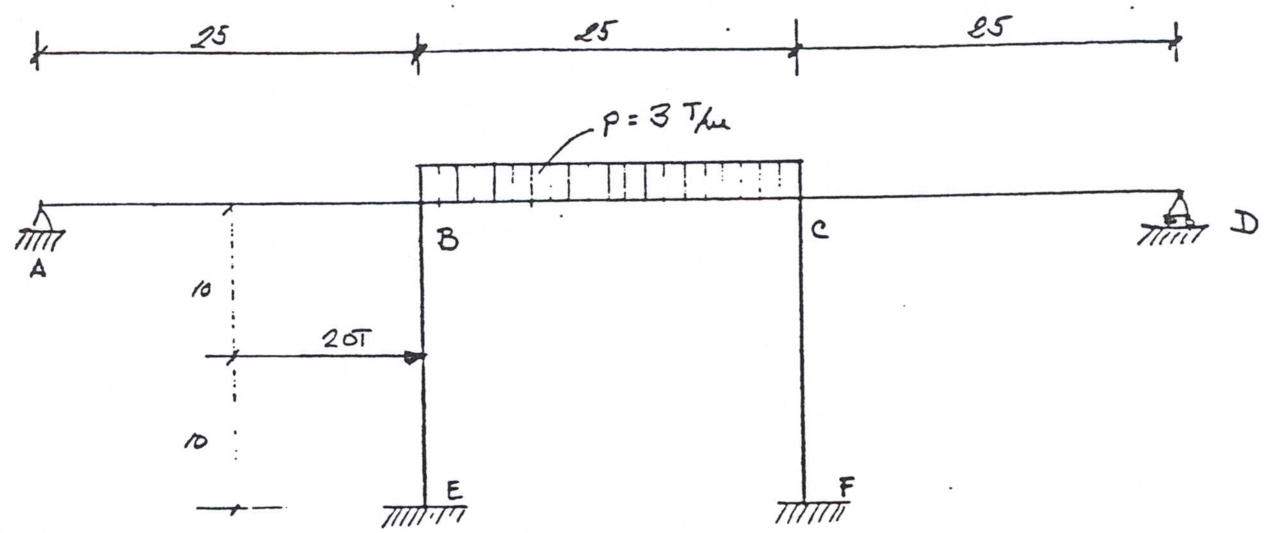
Asignatura: ANALISIS DE ESTRUCTURAS -METODOS NUMERICOS

Problema 2:

Calcular las Leyes de momentos flectores en el pórtico intraslacional de barras iguales de la figura, así como las reacciones y estudiar la deformada





Momentos de empotramiento:

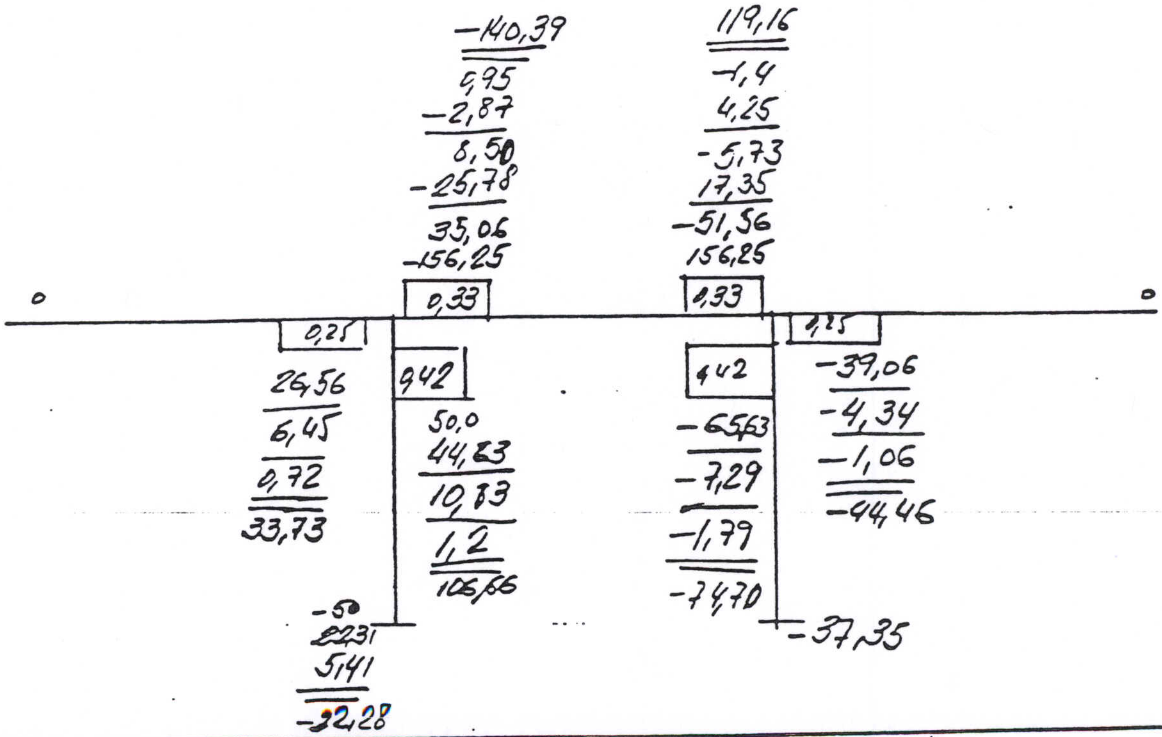
$$M_{BC} = -\frac{pL^2}{12} = -\frac{3 \times \overline{25}^2}{12} = -156,25 \text{ mT} \quad ; \quad M_{CB} = 156,25 \text{ mT}$$

$$M_{EB} = -\frac{PL}{8} = -\frac{20 \times 20}{8} = -50,0 \text{ mT} \quad ; \quad M_{BE} = 50,0 \text{ mT}$$

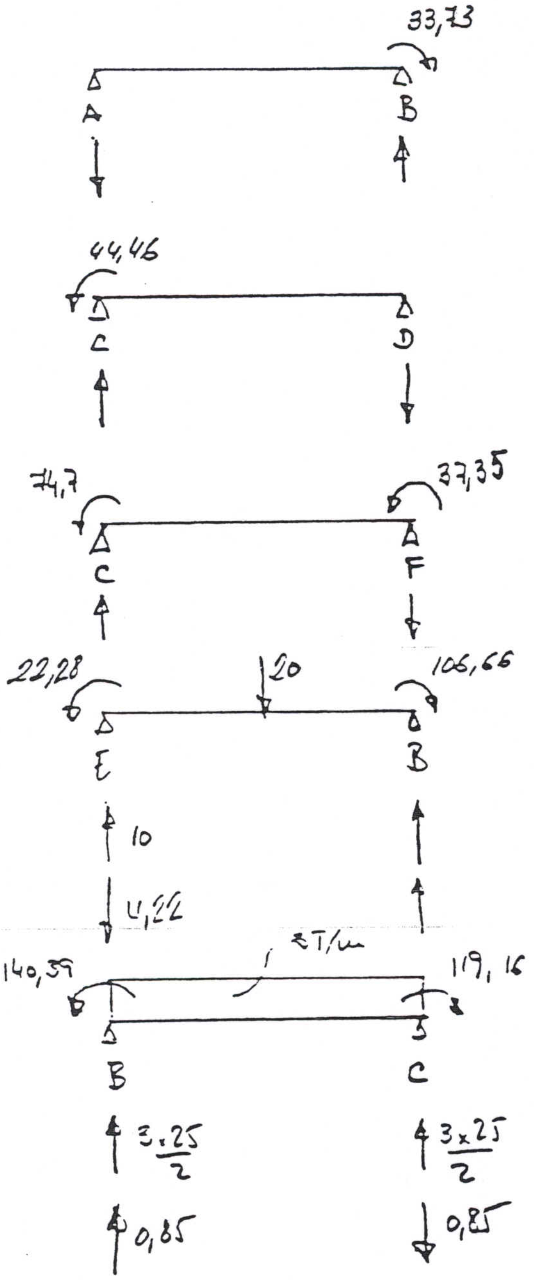
Rigideces:

$$K_{AB} = \frac{3EI}{25} \quad , \quad K_{BC} = \frac{4EI}{25} \quad , \quad K_{CD} = \frac{3EI}{25}$$

$$K_{BE} = K_{CF} = \frac{4EI}{20}$$



REACCIONES:



⟹ $R_A^V \downarrow = 1{,}35\ T$

Reacción

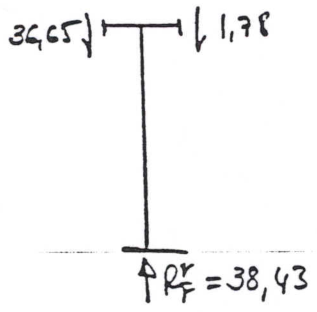
⟹ $R_D^V \downarrow = 1{,}78\ T$

⟹ $R_F^H \downarrow = 5{,}6\ T$

⟹ $R_E^H \uparrow = 5{,}78\ T$

⟹ $R_C \uparrow = 36{,}65\ T$

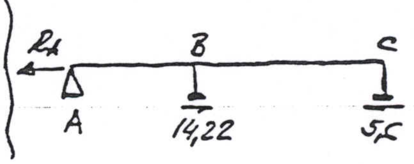
Reacción vertical en F:



Reacción vertical en E:

$R_E^V \uparrow = 3\times25 + 1{,}35 + 1{,}78 - 38{,}43 = 39{,}70$

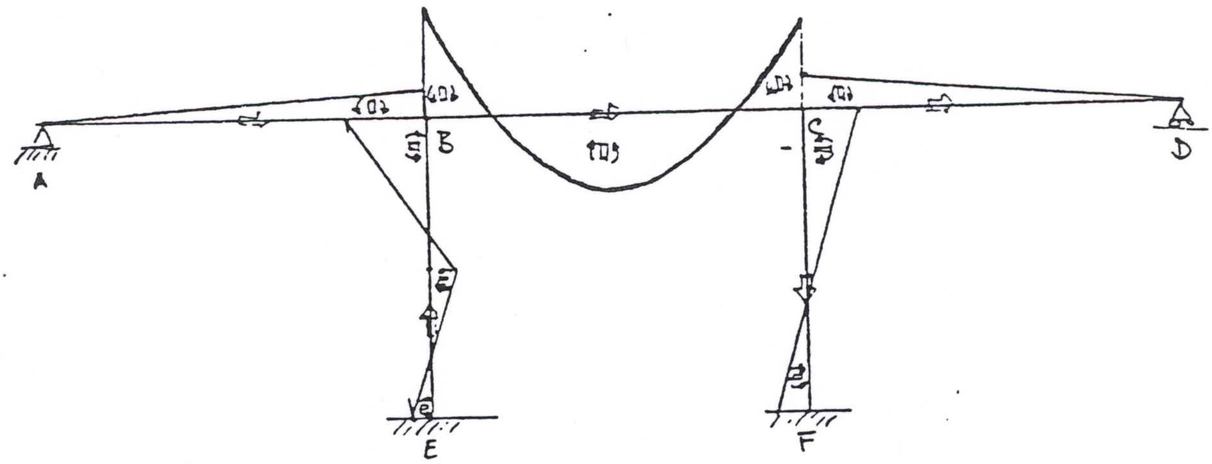
Reacción horizontal en A:

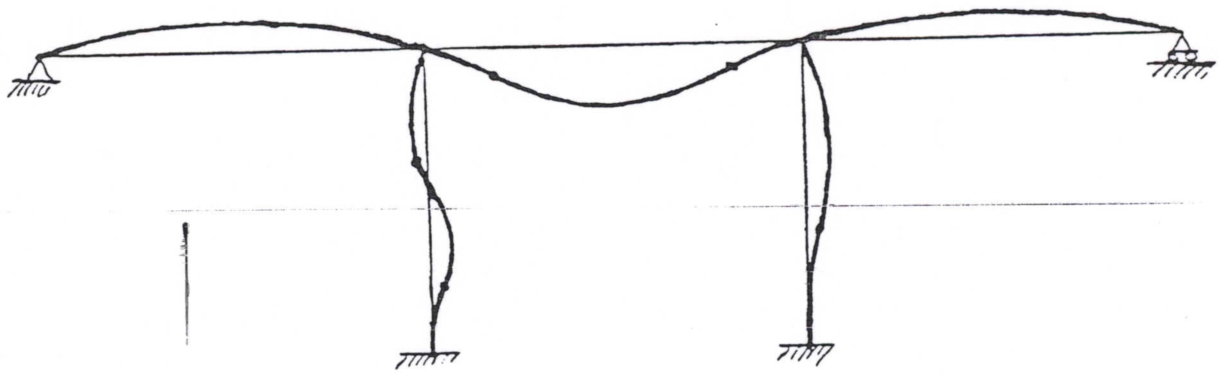


$R_A = 14{,}22 - 5{,}6 = 8{,}6$…

NOTA: Como comprob. Tomamos momentos respecto de E:

$$-22{,}28 - 1{,}35\times25 + 1{,}78\times50 - 38{,}43\times25 - 37{,}35 + 3\times\frac{25^2}{2} + 20\times10 - 8{,}62\times20 \simeq 0$$


A
B
C
D
E
F

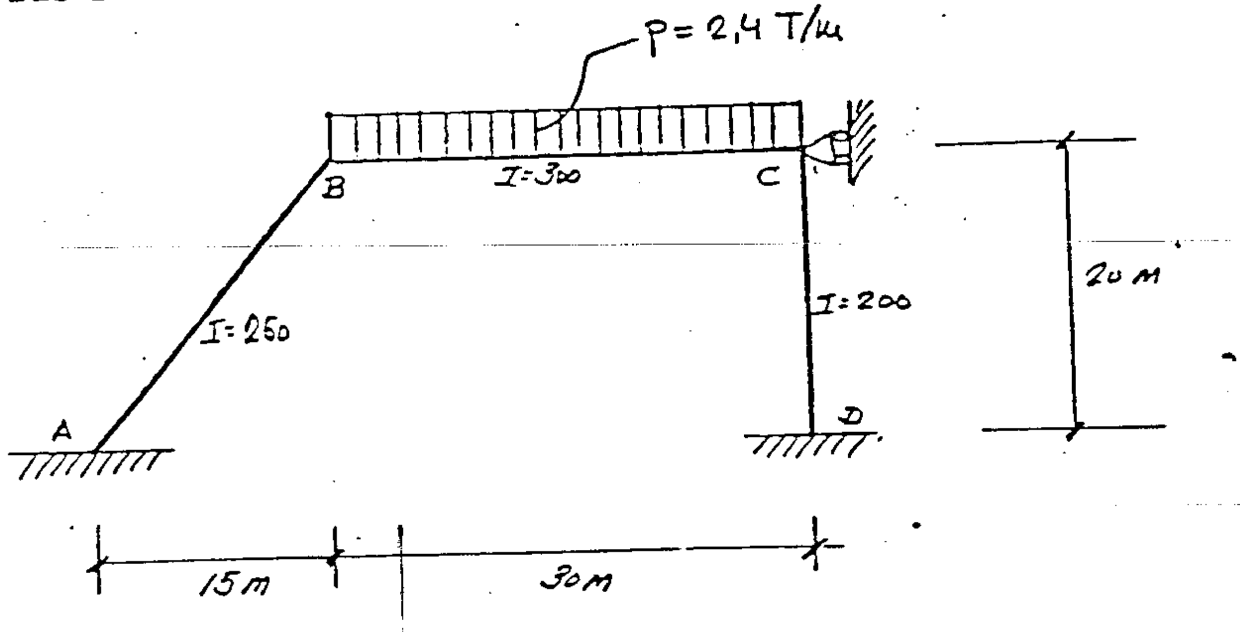


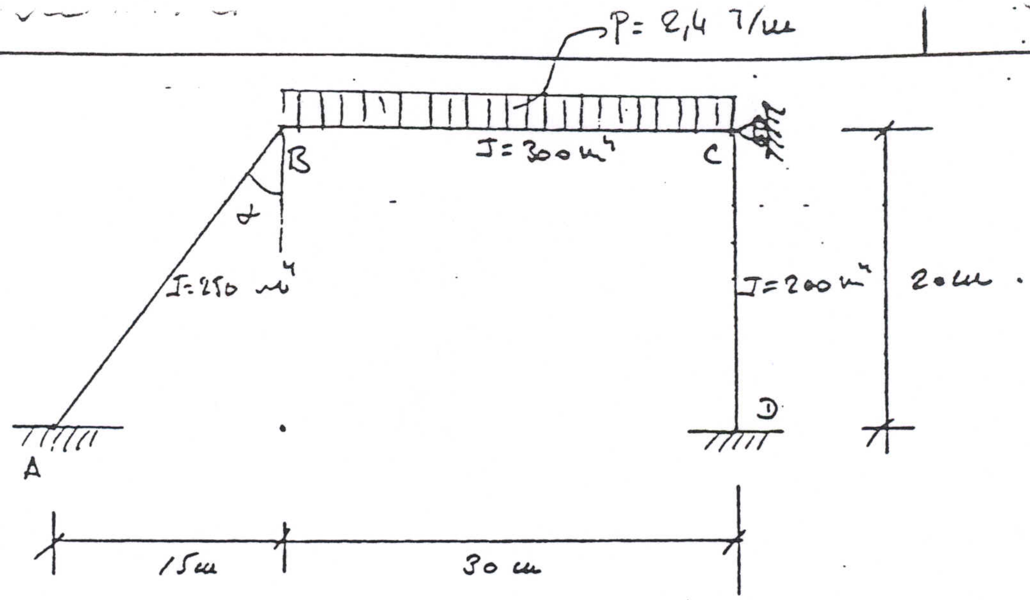
UNIVERSIDAD NACIONAL DE EDUCACION A DISTANCIA

Asignatura: ANALISIS DE ESTRUCTURAS-METODOS NUMERICOS

Probbema : 3

En la estructura intraslacional de la figura, determinar las leyes de momentos flectores, esfuerzos cortantes, axiles, así como las reacciones.



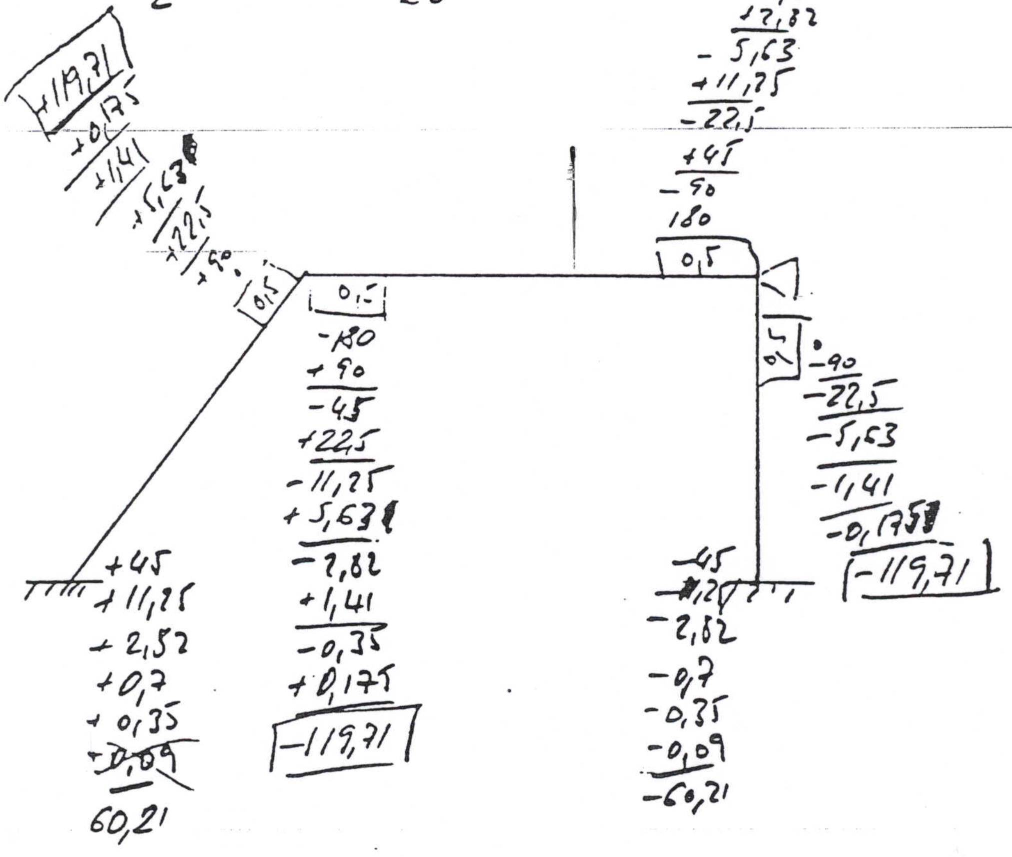


$$K_{AB} = \frac{4EI}{L} = \frac{4E\,250}{\sqrt{15^2+20^2}} = 40E$$

$$M_B = -\frac{pl^2}{12} = \frac{2{,}4 \times 30^2}{12} = 180$$

$$K_{BC} = \frac{4EI}{L} = \frac{4E\,300}{30} = 40E$$

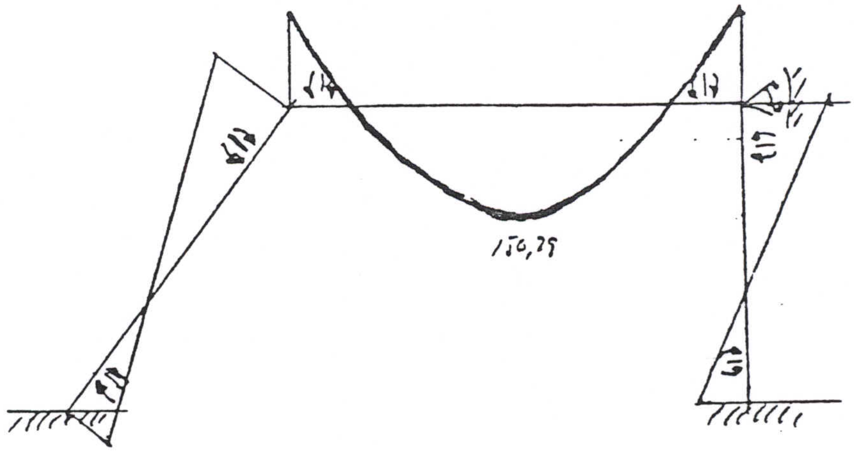
$$K_{CD} = \frac{4EI}{L} = \frac{4E\,200}{20} = 40E$$



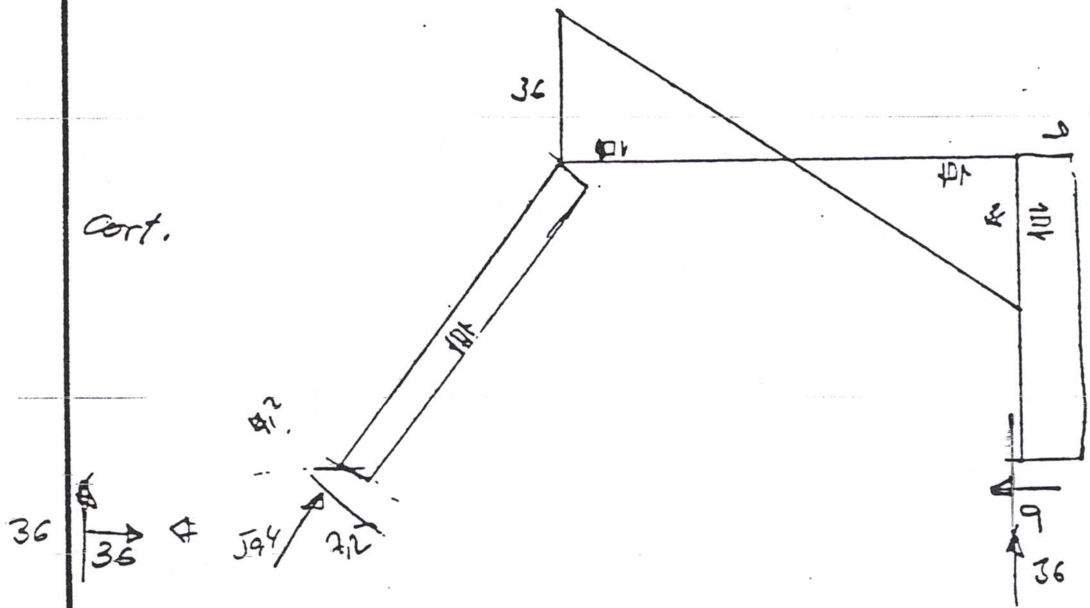
$$\alpha = 36{,}8699 \Rightarrow \begin{cases} \operatorname{sen} \alpha = 0{,}6 \\ \cos\alpha = 0{,}8 \end{cases}$$

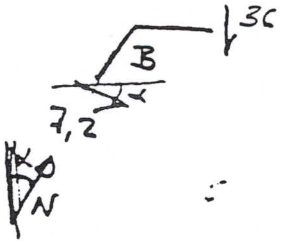
PROBLEMA. 6

Flectores.



Cort.

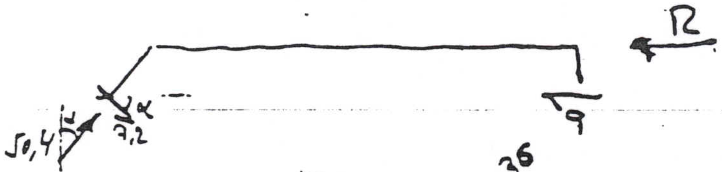




$$N \cos\alpha - 7{,}2 \operatorname{sen}\alpha - 36 = 0$$

$$N \times 0{,}8 = 7{,}2 \cdot 0{,}6 + 36$$

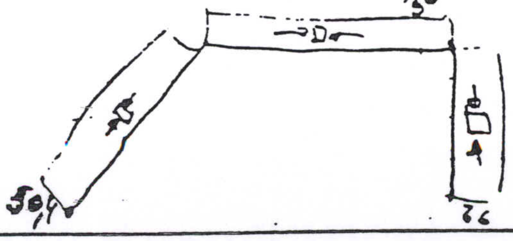
$$\boxed{N = 50{,}4}$$



$$R = 50{,}4 \times 0{,}6 + 7{,}2 \times 0{,}8 - 9 =$$

$$= \underline{27}$$

Axiles:



Asignatura : ANALISIS DE ESTRUCTURAS - METODOS NUMERICOS

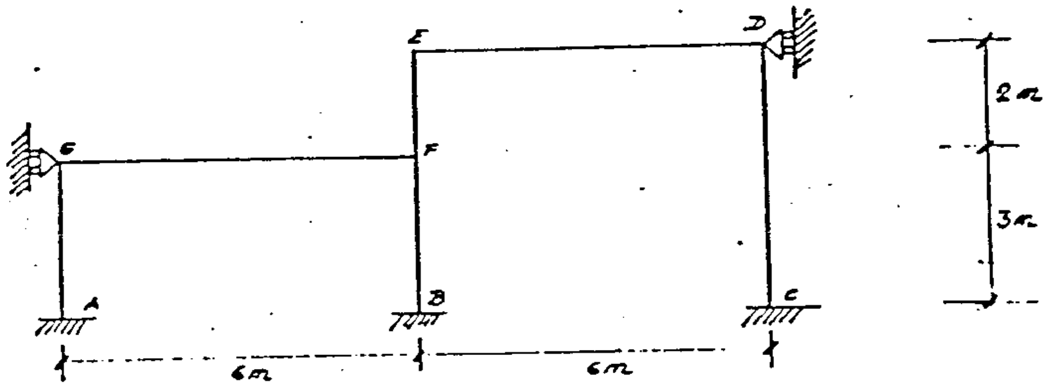
Problema : 4

- Hallar las reacciones horizontales en G y en D de la estructura representada en la figura, si el apoyo B sufre un descenso de 8 mm.

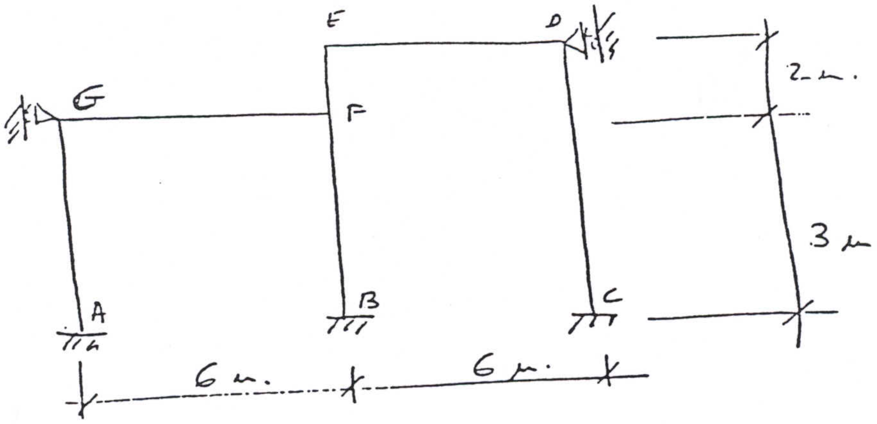
DATOS: E x I en las barras horizontales (DE y FG) = 9×10^3 Tm^2

E x I en las barras AG, BF y EF = $0,651 \times 10^3$ Tm^2

E x I en la barra CD = $2,667 \times 10^3$ Tm^2



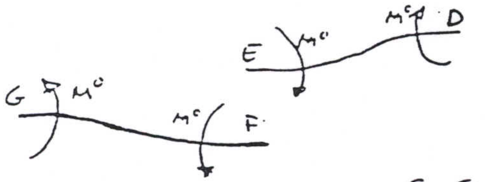
UD. 4 — Problema N° 4 Hoja 1/2 ✓



Barra DE y FG — $EI = 9 \cdot 10^3 \text{ t m}^2 = 9 \cdot 10^3 \text{ t} \cdot 100^2 \text{cm}^2 = 9 \cdot 10^7 \text{ t cm}^2$

" AG, BF y EF — $EI = 0{,}651 \cdot 10^3 \text{ t m}^2 = 0{,}651 \cdot 10^3 \text{ t} \cdot 10^4 \text{cm}^2 = 0{,}651 \cdot 10^7 \text{ t cm}^2$

" CD — $EI = 2{,}667 \cdot 10^3 \text{ t m}^2 = 2{,}667 \cdot 10^3 \text{ t} \cdot 10^4 \text{cm}^2 = 2{,}667 \cdot 10^7 \text{ t cm}^2$



$$M^\circ = \frac{6EI}{L^2}\Delta = \frac{6 \cdot 9 \cdot 10^7 \text{ t} \cdot \text{cm}^2}{(600)^2 \text{cm}^2} \cdot 0{,}8 \text{ cm} = 1200 \text{ t} \cdot \text{cm} = 12 \text{ t} \cdot \text{m}.$$

$$K_{GF} = 4\frac{EI}{\ell} = \frac{4 \cdot 9 \cdot 10^7 \text{ t cm}^2}{600 \text{ cm}} = 6 \cdot 10^5$$

$$K_{GA} = 4\frac{EI}{\ell} = \frac{4 \cdot 0{,}651 \cdot 10^7}{300} = 0{,}868 \cdot 10^5$$

$r_{GF} = 0{,}87$

$r_{GA} = 0{,}13$

$K_{FG} = 6 \cdot 10^5$

$K_{FB} = 0{,}868 \cdot 10^5$

$$K_{FE} = \frac{4 \cdot 0{,}651 \cdot 10^7}{200} = 1{,}302 \cdot 10^5$$

$r_{FG} = 0{,}73$

$r_{FB} = 0{,}11$

$r_{FE} = 0{,}16$

$K_{DE} = 6 \cdot 10^5$

$$K_{DC} = \frac{4 \cdot 2{,}667 \cdot 10^7}{500} = 2{,}1336 \cdot 10^5$$

$r_{DE} = 0{,}74$

$r_{DC} = 0{,}26$

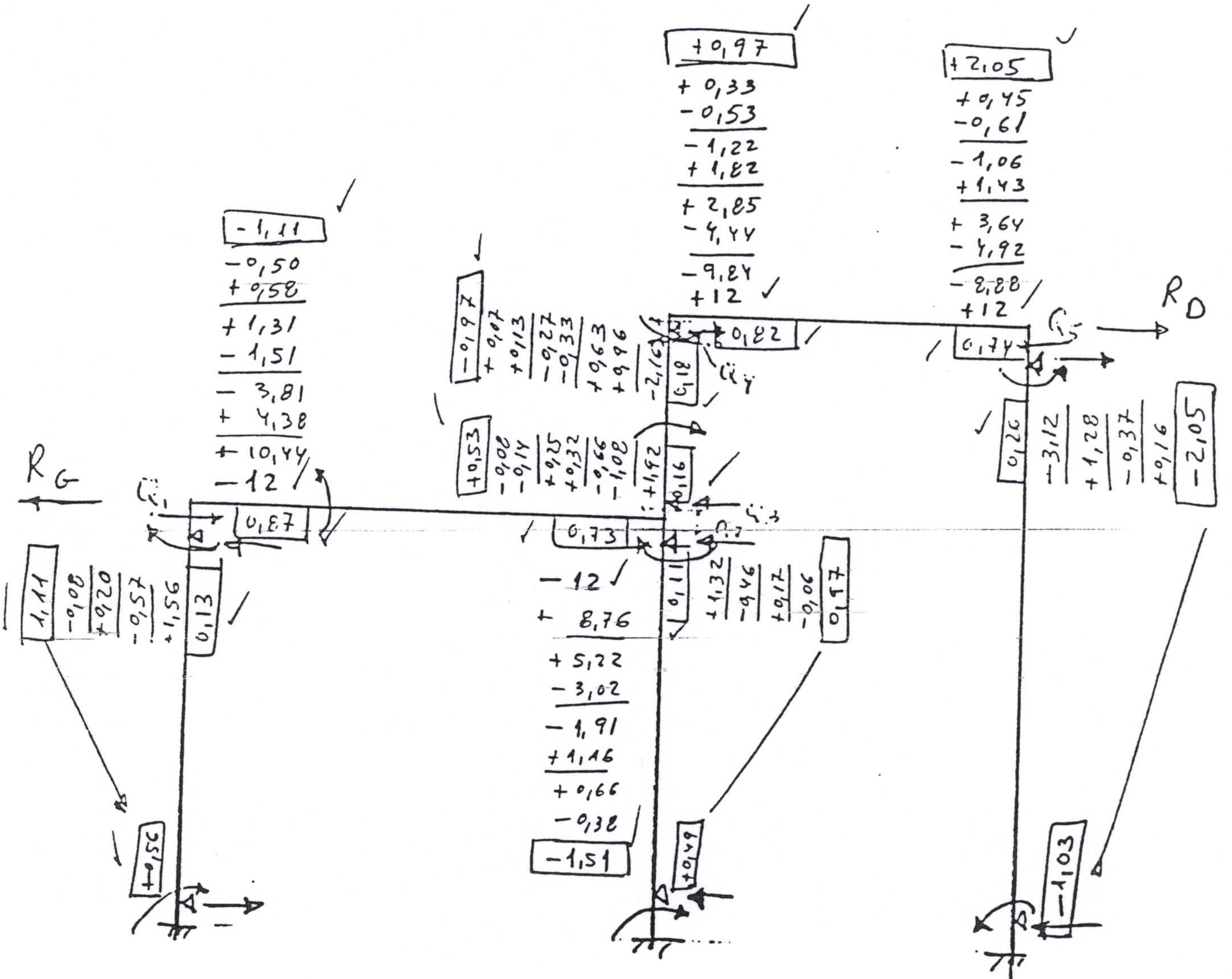
$K_{ED} = 6 \cdot 10^5$

$K_{EF} = 1{,}302 \cdot 10^5$

$r_{ED} = 0{,}82$

$r_{EF} = 0{,}18$

UD 4 — Problème 4 — Page 2/2



$$R_G = +Q_1 + Q_2 + Q_3 = + \frac{1{,}11 + 0{,}56}{3} + \frac{0{,}49 + 0{,}97}{3} + \frac{0{,}97 - 0{,}53}{2} = 1{,}26\ t = R_G$$

$$R_D = +Q_4 + Q_5 = + \frac{0{,}97 - 0{,}53}{2} + \frac{2{,}05 + 1{,}03}{5} = ~~1{,}76~~\ t = R_D$$

0,916

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Asignatura: ANALISIS DE ESTRUCTURAS-METODOS NUMERICOS

Problema: 5

Calcular las reacciones, leyes de momentos flectores, esfuerzos cortantes y axiles, en el pórtico de la figura. Sabiendo:

Las vigas AB, BH, CD, FG son de sección rectangular de 0,3x0,4 m. (ancho x alto) ; la viga CG, es también de sección rectangular de 0,3x0,5 m (ancho x alto).

La viga BC es de sección variable y de ella sabemos que: el momento de empotramiento en el nudo C al dar un momento M=4 en el extremo B es de MC=1,6 y el giro $\theta_B = 5 \times 10^{-4}$; y el momento de empotramiento en el nudo B MB es de 4 cuando aplicamos un momento M=5 en el extremo C.

Todas las longitudes están en metros

$E = 2 \times 10^6\ T/m^2$